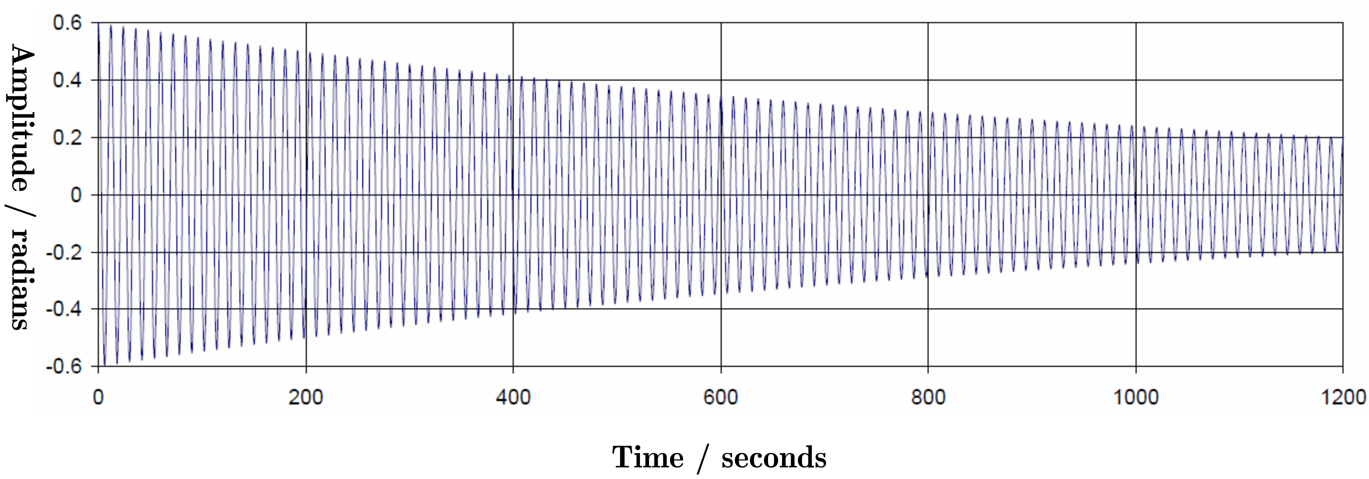
PHYSICS

*Answer **two** questions – you are advised to spend roughly 30 minutes on each question*

Question 1 Discuss the quality factor Q in damped oscillating systems; briefly explain how Q relates to relevant physical constants, and suggest how Q may be measured in two different ways. [4]

An antique clock keeps time by means of a torsional oscillator: the torsional bob has moment of inertia $I = 5.9 \times 10^{-5}$ kg m^2 and is suspended from a thin wire which provides a restoring force when the bob is rotated away from its equilibrium orientation. The system is very lightly damped. A free (undriven) oscillation of the rotation bob vs. time is shown in the graph below



Estimate angular frequency of the oscillation and the restoring torque per unit angular displacement (torsional spring constant) of the wire. [4]

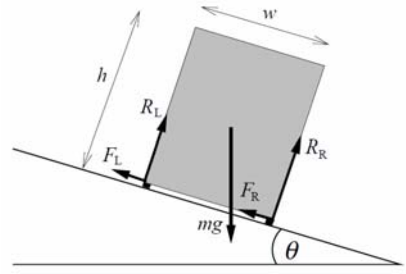
Estimate the quality factor of this oscillation. Briefly suggest why the quality factor might be a good measure of how accurate you might expect the clock to be when the mechanism is driven at its resonant frequency. [3]

Explain whether you would expect the clock to run fast or slow when it is placed at the centre of a light turntable which is free to rotate on a frictionless bearing. What additional piece(s) of information would you require to calculate how much faster or slower? [4]

> Newton's second law for a rotating body is $\tau = I\alpha$ where τ is the torque on the body, I is the body's moment of inertia and α is the body's angular acceleration
>
> A torsional spring (wire) exerts a torque $\tau = \kappa\theta$ on a body attached to it, where θ is the angular displacement of the body from equilibrium, and κ is the torsional spring constant of the wire

Question 2 State how the forces on a body affect both its linear and rotational motion, and give the conditions for a body to be in equilibrium. [3]

A uniform rectangular block of mass m, width w and height h, which has small ridges on two of its bottom edges, as shown, is placed on a rough plane inclined at an angle θ to the horizontal.



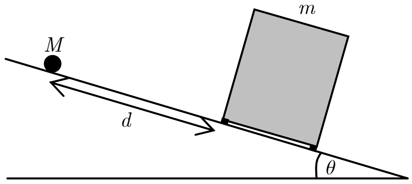
The forces acting on the block are its weight, the normal reaction forces R_L and R_R and the frictional forces F_L and F_R at the left and right ridges respectively, as shown. The coefficient of static friction between each of the ridges and the inclined plane is μ, and the masses and thicknesses of the ridges are negligible. How are the forces acting on the block related when the block is in equilibrium? [3]

Show that, as θ is increased,

(a) either the block will slide when $\tan\theta > \mu$ [*Hint*: consider the *total* frictional force and *total* reaction force on the block]

(b) or the block will tip over when $\tan\theta > w / h$ [4]

A point-like piece of ice of mass M is released from rest on the plane at a distance d from the block, slides freely on the surface, collides with the block and comes to rest as a result. The collision is observed to last 0.5 seconds (in other words, the ice touches the block, squishes slightly, and then comes to rest, and the whole process takes 0.5 seconds from the time the ice first touches the block to the time the ice comes to rest). The coefficient of friction between the ice and the surface is 0.

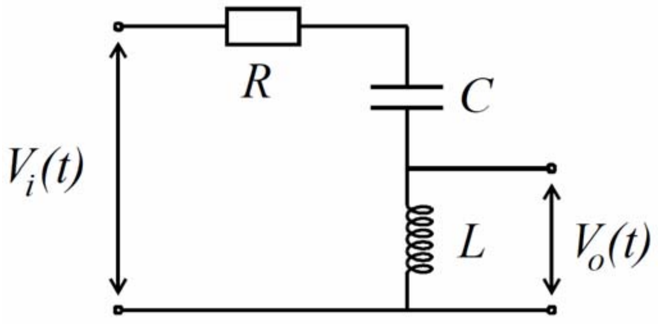


Given that the block does *not* slide as a result of the collision, show that [*Hint*: once again, consider the *total* frictional force on the block]

$$d < \frac{m^2 g\left(\mu\cos\theta - \sin\theta\right)^2}{8M^2\sin\theta}$$ [5]

Question 3 Define the term impedance as it relates to AC circuits and derive the expressions for the complex impedances of a capacitor and an inductor, $Z_C = 1/(i\omega C)$ and $Z_L = i\omega L$ respectively. [5]

In the circuit below, the voltage $V_i(t) = V_1 \operatorname{Re}\{e^{i\omega t}\}$ produces a current $I(t) = I_1 \operatorname{Re}\{e^{i(\omega t+\phi)}\}$. Determine I_1 and ϕ. [5]



The output voltage across the inductor is $V_0(t) = V_2 \operatorname{Re}\{e^{i(\omega t+\theta)}\}$. Determine V_2 and θ in terms of V_1, R, C, L and ω. Sketch graphs showing the dependence of V_2 and θ on ω. Indicate on your graph calculated values of V_2 and θ at $\omega = 0$, $\omega = (LC)^{-1/2}$ and the asymptotic values as $\omega \to \infty$. [5]