Exam 3 Review Questions

8.02 – Spring 2009

Question 1

Which of the following expressions *could* be a valid representation of the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields in an electromagnetic wave? (Remember that $\boldsymbol{i}\times\boldsymbol{j}=\boldsymbol{j}\times\boldsymbol{k}=\boldsymbol{k}\times\boldsymbol{i}=1$ but $\boldsymbol{j}\times\boldsymbol{i}=\boldsymbol{k}\times\boldsymbol{j}=\boldsymbol{i}\times\boldsymbol{k}=-1$)

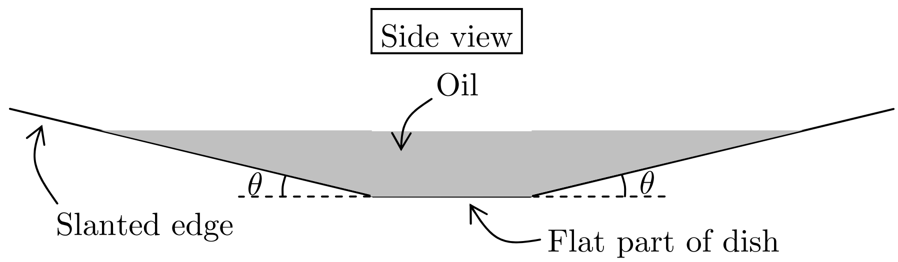
1. $\boldsymbol{E}=E_0\cos\left(ky-\omega t\right)\boldsymbol{j}$ $\boldsymbol{B}=B_0\cos\left(ky-\omega t\right)\boldsymbol{k}$
2. $\boldsymbol{E}=2\cos\left(kx-\omega t\right)\boldsymbol{j}$ N/C $\boldsymbol{B}=5\cos\left(kx-\omega t\right)\boldsymbol{k}$ T
3. $\boldsymbol{E}=E_0\cos\left(kx-\omega t\right)\boldsymbol{k}$ $\boldsymbol{B}=B_0\cos\left(kx-\omega t\right)\boldsymbol{j}$
4. $\boldsymbol{E}=E_0\cos\left(ky+\omega t\right)\boldsymbol{i}$ $\boldsymbol{B}=B_0\cos\left(ky+\omega t\right)\boldsymbol{k}$
5. $\boldsymbol{E}=E_0\cos\left(kx-\omega t\right)\boldsymbol{j}$ $\boldsymbol{B}=B_0\cos\left(ky-\omega t\right)\boldsymbol{k}$
6. $\boldsymbol{E}=E_0\cos\left(kx-\omega t\right)\boldsymbol{k}$ $\boldsymbol{B}=B_0\cos\left(kx-\omega t\right)\boldsymbol{k}$
7. $\boldsymbol{E}=E_0\cos\left(kx-\omega t\right)\boldsymbol{j}$ $\boldsymbol{B}=B_0\cos\left(kx+\omega t\right)\boldsymbol{k}$

The answer is 4.

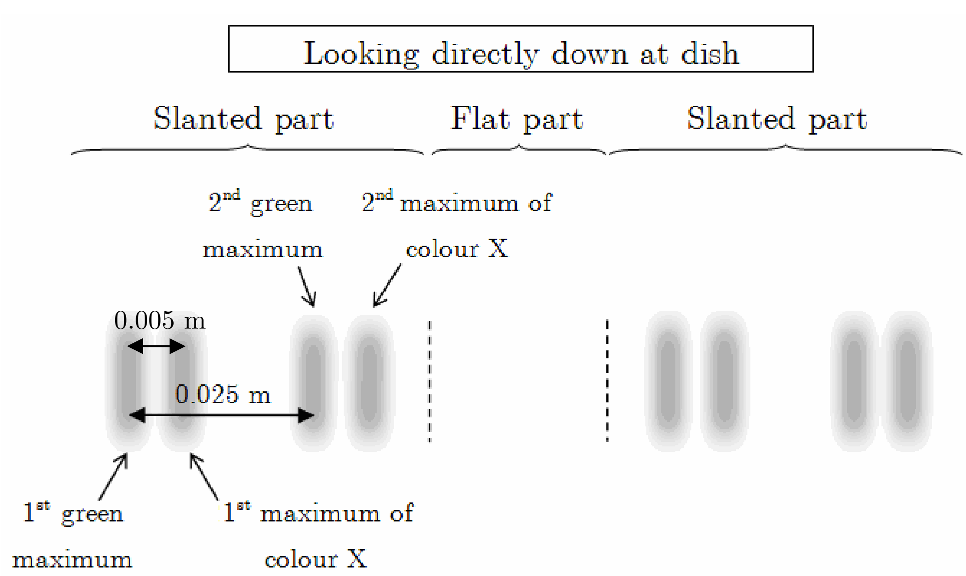
1. …is not valid because the direction of propagation is $+y$, but the $\boldsymbol{E}$ fields are along the $+y$ direction. The wave is therefore not transverse.
2. …is not valid because the $\boldsymbol{B}$ field is larger than the $\boldsymbol{E}$ field.
3. …is not valid because the wave propagates in the $+x$ direction, but the Poynting vector is in the $\boldsymbol{S}=\left|\boldsymbol{E}\times\boldsymbol{B}\right|=\left|\boldsymbol{k}\times\boldsymbol{j}\right|=-\boldsymbol{i}$ direction.
4. …is valid!
5. …is not valid because the $\boldsymbol{E}$ and $\boldsymbol{B}$ field depend on different spatial coordinates (x and y).
6. …is not valid because the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields point in the same direction, not orthogonal to each other.
7. …is not valid because the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields travel in different directions. The $\boldsymbol{E}$ field is travelling forwards, and the $\boldsymbol{B}$ field is travelling backwards.

Question 2

I have, in my kitchen, a very long serving dish, with a very narrow flat base and large *slowly* slanting sides. Some 8.02 students, wanting to take revenge for how hard the last PSet was, came along and filled it with diesel oil. Here's a side view of what it looked like:



When I looked directly down at the dish, I saw a beautiful colourful pattern. Here's a simplified sketch of what I saw in which I only considered two of the many colours visible (not to scale):



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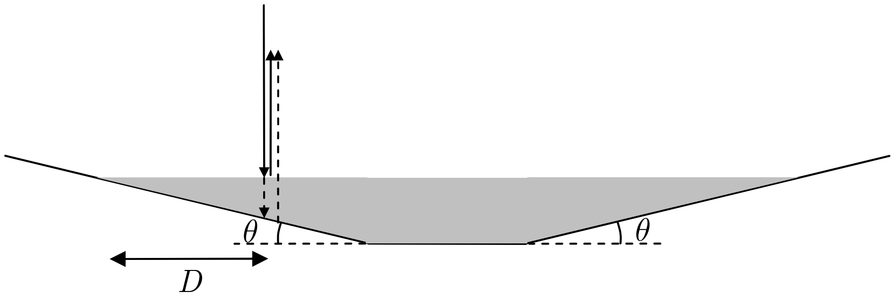
Why did interference occur? Here's a sketch explaining it:



A beam that falls on the oil surface can either reflect straight back (solid arrow) or go through the oil, reflect from the dish and then back up through the oil (dotted arrow). Since the dotted beam has travelled longer than the solid beam, they interfere.

You may find the following data useful in answering this problem

Colour	Wavelength (nm)
Red	650
Orange	600
Green	500
Violet	400
Note: 1 nm = 10^{-9} m	

Part A

Why did I only see fringes in the slanted parts of the dish, and not in the flat parts?

The path difference between the two interfering beams was constant over the flat part of the dish. Therefore, no fringes appeared – the intensity was constant over the whole flat part.

Part B

In the last diagram above, in which a beam enters the oil at a distance D from the edge of the oil, find the path difference between the two beams. Give your answer in terms of D and θ. [*Hint*: the dotted beam travels through the oil **twice** – once on its way down, and once on its way back up]

Using simple trigonometry, distance from the top of the oil to the dish at that point is

$$h = D\tan\theta$$

The path difference is *twice* this distance, because the beam goes down and back up, and so

$$\boxed{\delta = 2D\tan\theta}$$

Part B

For this part, consider the *green* fringes only, and find $\tan\theta$.

We know that at each of the maxima, the path difference is an integer number of wavelengths. At the first maximum, we know that it's exactly one wavelength, and at the second maximum, we know that it's exactly two wavelengths:

$$\delta_{\text{first maximum}} = 2D_{\text{first maximum}}\tan\theta = \lambda$$
$$\delta_{\text{second maximum}} = 2D_{\text{second maximum}}\tan\theta = 2\lambda$$

But we know from the diagram that

$$D_{\text{first maximum}} - D_{\text{second maximum}} = 0.025\,\text{m}$$

Therefore, using the data in the table

$$\left(0.025\,\text{m}\right)\tan\theta = \frac{500\,\text{nm}}{2}$$
$$250\times10^{-4}\tan\theta = \frac{500\times10^{-9}}{2}$$
$$\boxed{\tan\theta = 10^{-5}}$$

[My bowl really wasn't very slanted at all!]

Part C

Why did different colours produce maxima at different places?

Different colours have different wavelengths, and so will interfere constructively at different path differences, which correspond to different locations on the surface of the oil.

Part E

What was colour X?

Let λ_X be the wavelength of colour X, and λ_G be the colour of green light. We know (since we are considering *first* maximum) that

$$2D_{\text{first green}} \tan\theta = \lambda_{\text{green}}$$
$$2D_{\text{first X}} \tan\theta = \lambda_X$$

Furthermore, we know from the diagram that

$$D_{\text{first X}} - D_{\text{first green}} = 0.005\,\text{m}$$

Therefore

$$2\left(0.005\,\text{m}\right)\tan\theta = \lambda_X - \lambda_G$$
$$\lambda_X = 2\left(0.005\,\text{m}\right)\tan\theta + \lambda_G$$

Using our value of $\tan\theta$ from the previous part and of λ_G from the table, we get

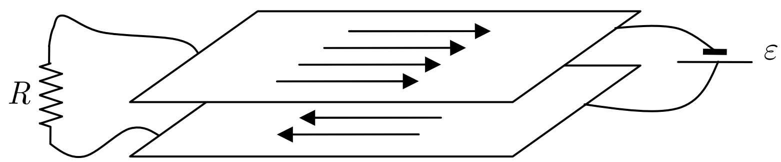
$$\lambda_X = 2\left(0.005\,\text{m}\right)\left(10^{-5}\right) + \left(500\times10^{-9}\right)$$
$$\lambda_X = 10\times10^{-8} + \left(500\times10^{-9}\right)$$
$$\boxed{\lambda_X = 600\,\text{nm}}$$

Looking at the table above, we see that colour X is in fact **orange**.

Question 3

Consider the following setups:

- A parallel-plate capacitor with circular plates of radius R is **charging** constantly (ie: a constant current I is flowing through it and *increasing* the charge on the plates).
- A solenoid with turns-per-unit-length n which is **discharging** (ie: the current through it is decreasing at a constant rate).
- A cylindrical resistor of length ℓ, radius a and resistance R through which a current I is flowing, and which is dissipating energy.
- Two large parallel square plates of side length L a small distance d apart with *no* resistance carrying current to a resistor and back



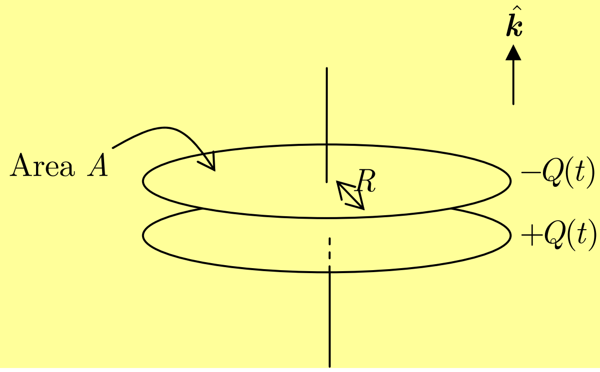
For each of these cases, calculate

- The $\boldsymbol{E}$ field
- The $\boldsymbol{B}$ field
- The Poynting vector, $\boldsymbol{S}$

In each case, check whether the answer your obtain for the Poynting vector makes sense.

THE CHARGING CAPACITOR

Let's imagine that the capacitor is charging as follows:



In this case, the "obvious" field is the **electric field**. If the charge on the plates is Q at any given time, then we've worked out a million times before that

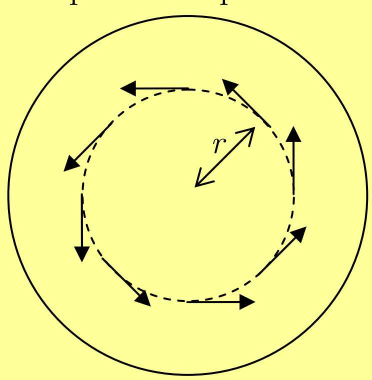
$$\boldsymbol{E}(t) = \frac{Q(t)}{A\varepsilon_0}\hat{\boldsymbol{k}}$$

$$\boxed{\boldsymbol{E}(t) = \frac{Q(t)}{\pi R^2\varepsilon_0}\hat{\boldsymbol{k}}}$$

Now, what about the **magnetic field**? There are two possible sources (a) free currents and (b) displacement currents (changing electric fields). Obviously, in this case, only (b) applies. And so

$$\oint_C \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{l} = \mu_0\varepsilon_0 \frac{\partial \Phi_{E,S}}{\partial t}$$

The question is, what surface S and curve C should we use to evaluate this integral? By symmetry in the problem, it looks like the B field should only depend on distance from the centre of the capacitor, and should be tangential everywhere. Looking from the top of the capacitor:



How do I know the direction? Well, the field is going **up** and is **increasing**, and so $\partial\Phi / \partial t$ is also pointing **up**. Thus, by the right-hand-rule, the field is as above.

In any case – it seems like a concentric circle of radius r works nicely – I've shown it dotted in the diagram above.

Doing the integral:

$$2\pi r B = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\pi r^2 E\right)\hat{\boldsymbol{\theta}}$$

$$2\pi r B = \mu_0 \varepsilon_0 \pi r^2 \frac{\partial E}{\partial t}\hat{\boldsymbol{\theta}}$$

$$B = \frac{1}{2}\mu_0 \varepsilon_0 r \frac{\partial E}{\partial t}\hat{\boldsymbol{\theta}}$$

Using our expression for E above, we get

$$B = \frac{1}{2}\frac{\mu_0 \varepsilon_0 r}{\pi R^2 \varepsilon_0}\frac{\partial Q}{\partial t}\hat{\boldsymbol{\theta}}$$

$$\boxed{\boldsymbol{B} = \left(\frac{\mu_0 r}{2\pi R^2} I\right)\hat{\boldsymbol{\theta}}}$$

Note that both the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields depend on the radius r from the centre.

Finally, the Poynting vector!

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}$$

$$\boldsymbol{S} = \frac{1}{\mu_0}\frac{Q(t)}{\pi R^2 \varepsilon_0}\frac{\mu_0 r}{2\pi R^2} I\left(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{\theta}}\right)$$

$$\boxed{\boldsymbol{S} = \frac{1}{2\left(\pi R^2\right)^2 \varepsilon_0} r I Q(t)\left(-\hat{\boldsymbol{r}}\right)}$$

This is pointing *inwards*, towards the *inside* of the capacitor. This makes sense, because the capacitor is *charging*, and so energy is flowing *into it*.

Let's see if it makes sense quantitatively. To find the *total power* flowing into the capacitor, we need to integrate $\boldsymbol{S}$ over the area through which it is flowing at the outmost radius. In this case, it's flowing through the sides of the capacitor. We need

$$P = \iint_{\text{sides of capacitor}} \boldsymbol{S}\cdot \mathrm{d}\boldsymbol{A}$$

Since the vector is constant over that area, we simply multiply the vector by that area ($= 2\pi R d$). We get

$$P = \frac{2\pi R d}{2\left(\pi R^2\right)^2 \varepsilon_0} RIQ(t)$$

$$\boxed{P = \frac{dIQ}{\pi R^2 \varepsilon_0}}$$

Now, consider the total energy stored in the capacitor. We only consider the *electric* energy, because that's the only field that will *store* energy in the capacitor. We get an electric energy density of

$$U_E = \frac{1}{2}\varepsilon_0 \frac{Q^2}{\left(\pi R^2\right)^2 \varepsilon_0^2} = \frac{Q^2}{2\left(\pi R^2\right)^2 \varepsilon_0}$$

The volume in which this exists is $\pi R^2 d$, and so

$$U = \frac{\pi R^2 d Q^2}{2\left(\pi R^2\right)^2 \varepsilon_0} = \frac{dQ^2}{2\pi R^2 \varepsilon_0}$$

Finally, differentiating with respect to time

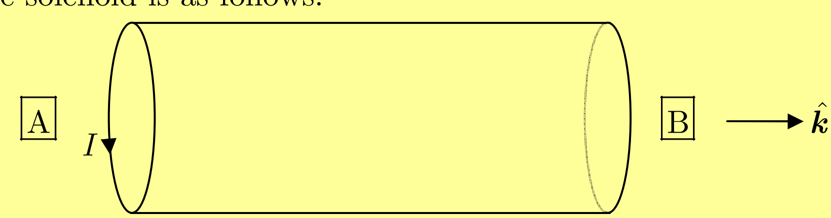
$$P = \frac{\mathrm{d}U}{\mathrm{d}t} = \frac{dQ}{\pi R^2 \varepsilon_0}\frac{\mathrm{d}Q}{\mathrm{d}t}$$

$$\boxed{P = \frac{dQI}{\pi R^2 \varepsilon_0}}$$

Which is precisely the same as the expression we calculated for the power flowing *into* the capacitor.

THE DISCHARGING SOLENOID

Imagine the solenoid is as follows:



As usual, let's first ask – which is the obvious field in this case? Clearly, it's the magnetic field, because its produced by the current in the coils of the solenoid.

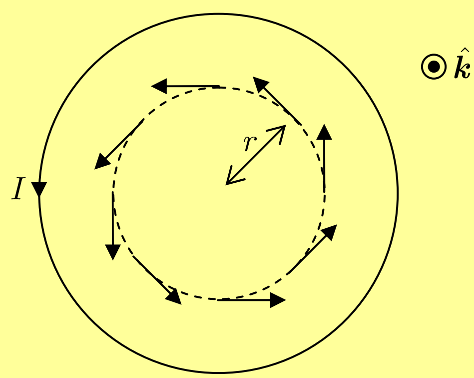
As you've worked out a million times before, the magnetic field in a solenoid carrying a current I is

$$\boxed{\boldsymbol{B} = \mu_0 n I \ \hat{\boldsymbol{k}}}$$

What the bout the electric field? Clearly, there are no charges in sight (remember: the charges moving in the wires are cancelled by fixed positive ions, so the wires are electrically neutral). Therefore, it's a change in magnetic field that is causing the electric field, and we need Faraday's Law

$$\oint_C \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{\partial \Phi_{B,S}}{\partial t}$$

Once again, we ask – which curve should we use to evaluate the law? By symmetry, the field in the solenoid should look something like this (looking down end **B** of the solenoid):



Why this direction? Well, the field is in the $+\hat{\boldsymbol{k}}$ direction, but it's **decreasing**, and so $\partial \Phi_B / \partial t$ is in the $-\hat{\boldsymbol{k}}$ direction. Therefore, $-\partial \Phi_B / \partial t$ is in the $\hat{\boldsymbol{k}}$ direction, and this gives the field above.

It looks like a good loop to choose is therefore a concentric circle of radius r (shown dotted in the diagram).

Faraday's Law then becomes

$$2\pi r E = \pi r^2 \frac{\partial B}{\partial t} \hat{\boldsymbol{\theta}}$$

$$E = \frac{r}{2} \frac{\partial B}{\partial t} \hat{\boldsymbol{\theta}}$$

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Feeding in our expression for B

$$\boxed{\boldsymbol{E} = \frac{r}{2}\mu_0 n \frac{\partial I}{\partial t}\hat{\boldsymbol{\theta}}}$$

The Poynting vector is then given by

$$\boldsymbol{S} = \frac{1}{\mu_0}\frac{r}{2}\mu_0 n \frac{\partial I}{\partial t}\mu_0 n I\left(\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{k}}\right)$$

$$\boxed{\boldsymbol{S} = \frac{1}{2}\mu_0 r n^2 I \frac{\partial I}{\partial t}\hat{\boldsymbol{r}}}$$

It looks like power is flowing *out* of the solenoid, which makes perfect sense because it's discharging.

Note, once again, that this Poynting vector depends on the radius.

Once again, let's see if the total amount of power flowing out of our solenoid is consistent with our expression for the energy in an solenoid.

To find the total power flowing out, we need to integrate the Poynting vector over the outmost area through which it flows.

$$P = \iint \boldsymbol{S} \cdot \mathrm{d}A$$

In this case, the Poynting vector is constant over that outer surface, and so we simply need to multiply it by the area of that surface ($= 2\pi R\ell$, where R is the radius of the solenoid). We get

$$P = \frac{1}{2}\mu_0 R n^2 I \frac{\partial I}{\partial t} 2\pi R\ell$$

$$\boxed{P = \mu_0 \pi R^2 \ell n^2 I \frac{\partial I}{\partial t}}$$

Now, consider the magnetic energy stored in the solenoid (again, we ignore electric energy, because only magnetic energy will be *stored*). The energy density is

$$U_B = \frac{1}{2\mu_0}B^2 = \frac{\mu_0 n^2 I^2}{2}$$

This is, mercifully, constant. So multiplying by the volume of the solenoid, we find that the total stored energy is

$$U = \pi R^2 \ell \frac{\mu_0 n^2 I^2}{2}$$

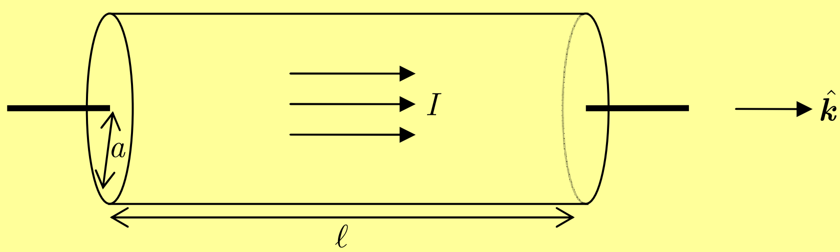
Differentiating with respect to time

$$\boxed{P = \frac{\mathrm{d}U}{\mathrm{d}t} = \pi R^2 \ell \mu_0 n^2 I \frac{\mathrm{d}I}{\mathrm{d}t}}$$

Precisely as above.

THE RESISTOR

Imagine our resistor is a cylinder, with a current flowing through it



In this case, both the $\boldsymbol{E}$ field and the $\boldsymbol{B}$ field are "obvious". Let's look at each one:

- There must be an $\boldsymbol{E}$ field *parallel* to the resistor, in the $\hat{\boldsymbol{k}}$ direction, to "push" the electrons along through the resistor, which is resisting the motion of the electrons. This comes from the potential difference $V = IR$ across the resistor. It's pretty obvious from $V = Ed$ that

$$\boxed{\boldsymbol{E} = \frac{IR}{\ell}\hat{\boldsymbol{k}}}$$

- There must be an $\boldsymbol{E}$ field produced by the current flowing. We can use a circular Amperian loop at a distance r from the centre of the cable:

$$\oint_C \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \mu_0 I_{\text{enc}}$$
$$2\pi r B = \mu_0 I_{\text{enc}}$$

  Assuming the current is uniformly distributed through the wire, we get

$$2\pi r B = \mu_0 \frac{I\pi r^2}{\pi a^2}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi a^2} r\,\hat{\boldsymbol{\theta}}}$$

And finally, the Poynting vector

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}$$

$$\boldsymbol{S} = \frac{1}{\mu_0}\frac{IR}{\ell}\times\frac{\mu_0 I}{2\pi a^2} r\left(\hat{\boldsymbol{k}}\times\hat{\boldsymbol{\theta}}\right)$$

$$\boxed{\boldsymbol{S} = \frac{I^2 R}{2\pi a^2 \ell} r\left(-\hat{\boldsymbol{r}}\right)}$$

Once again, the Poynting vector varies depending on radius.

The direction in this case is slightly more difficult to understand – why is energy flowing **into** the resistor? Surely it should be flowing *out*, because energy is dissipating as heat!

The key point to understand here is that the Poynting vector talks of **electromagnetic** energy. True, energy is flowing out of the resistor as heat, but where does that energy come from? It turns out that it comes from the electromagnetic energy that is flowing *into* the resistor, as we've just calculated.

To make sure both results are consistent, let's integrate the Poynting vector over the *entire surface* of the resistor at a radius $r = a$, to find the *total power* coming into the resistor (if we did it at another radius, we wouldn't find the power coming into the *whole* resistor).

$$P = \iint \boldsymbol{S}\cdot \mathrm{d}\boldsymbol{A}$$

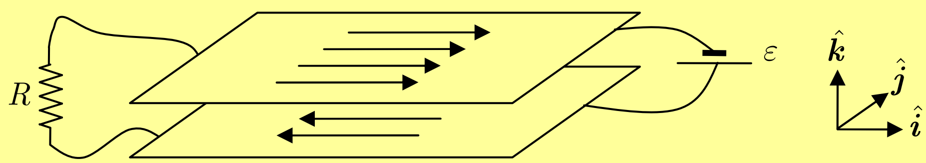
Since $\boldsymbol{S}$ is constant at the surface (which is at constant r), we simply need to multiply $\boldsymbol{S}$ by the relevant area (which is the curved surface area of the resistor, $2\pi a\ell$), and we get

$$P = \frac{I^2 R a}{2\pi a^2 \ell} 2\pi a \ell = I^2 R$$

Which is indeed the power dissipated in a resistor.

THE TWO PLATES

First, let's ask what the "obvious" field is. In this case, both are obvious – an electric field is caused by the potential difference between the plates, and a magnetic field is caused by the current.



Let's begin with the electric field. Trivially, $V = Ed$, and so

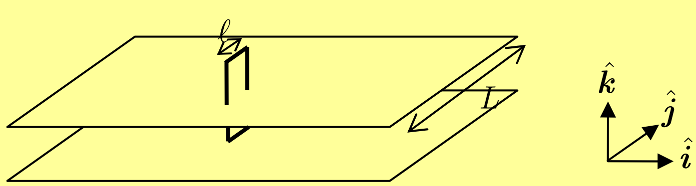
$$\boxed{\boldsymbol{E} = \frac{\varepsilon}{d}\hat{\boldsymbol{k}}}$$

Note: it is crucial to realise that there is *no* $\boldsymbol{E}$ fields along the plates here, because there is no resistant. There's no need for anything to "push" the electrons along.

For the magnetic field, we need to use Ampere's Law. A few points:

- First, we note that there'll only be a magnetic field **between** the plates – outside the plates, the currents cancel.
- Second, we note that the field will be in the $+\hat{\boldsymbol{j}}$ direction – use the right hand rule on the currents to see why.

We can use following Amperian loop:



We have

$$\oint_C \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{l} = \mu_0 I_{\text{enc}}$$

Let's look at each side:

- The left-hand-side is clearly

$$\oint_C \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \ell B$$

  Because the only side of the loop that contributes is the one in between the plates.

- The right-hand-side is a bit more tricky. Let's imagine a total current I is flowing through the plates. The current "spreads out" over the length of the plates, so the current per unit length is I/L. This means that the total current flowing through the loop is

$$I_{\text{enc}} = \frac{\ell I}{L}$$

Using Ampere's Law, we then get

$$\ell B = \mu_0 \frac{\ell I}{L}$$

$$\boxed{\boldsymbol{B} = \frac{\mu_0 I}{L}\hat{\boldsymbol{j}}}$$

Finally, the Poynting vector:

$$\boldsymbol{S} = \frac{1}{\mu_0}\frac{\varepsilon}{d}\frac{\mu_0 I}{L}\left(\hat{\boldsymbol{k}} \times \hat{\boldsymbol{j}}\right)$$

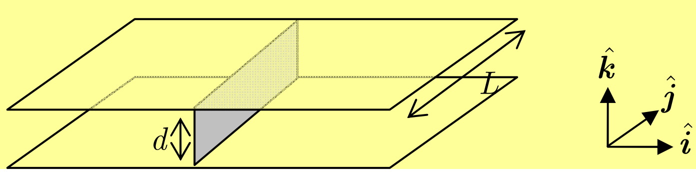
$$\boxed{\boldsymbol{S} = \frac{\varepsilon I}{dL}\left(-\boldsymbol{i}\right)}$$

As expected, power flows in the $-\boldsymbol{i}$ direction, from the battery to the resistor.

We can gain even further insight by finding the *total* flux from the batter to the resistor. It is

$$P = \iint \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{A}$$

Because the Poynting vector is constant, we just need to multiply by the area through which the Poynting vector flows, which I've sort of shaded in grey in this diagram:



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We then get

$$P = \frac{\varepsilon I}{dL} Ld$$

$$\boxed{P = \varepsilon I}$$

This makes complete sense – the power dissipate through a resistor is equal to VI, and this is indeed what we get for the power that flows from the battery to the resistor.

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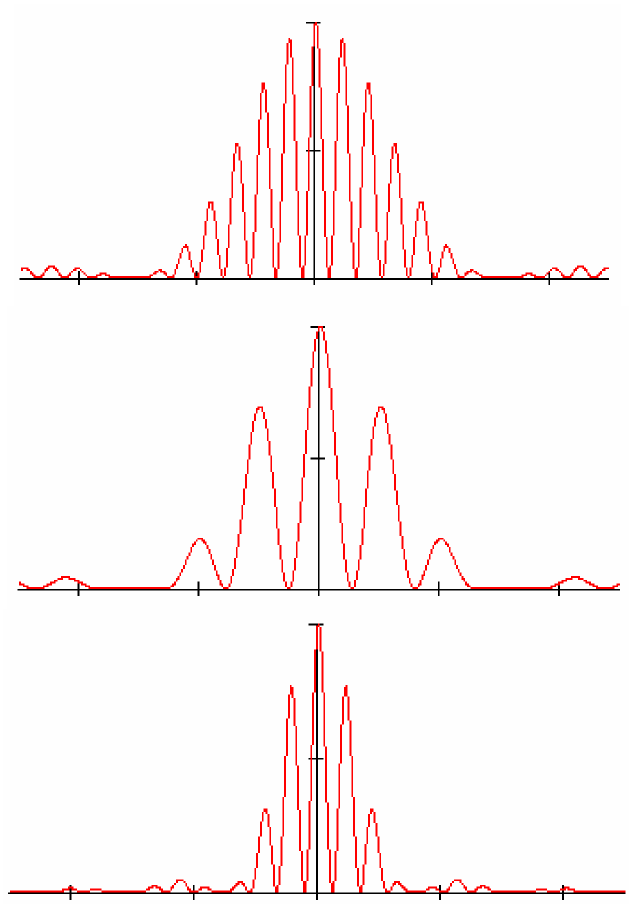
<u>Question 4</u>

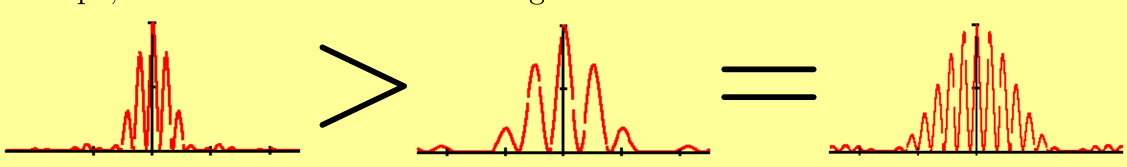
Look at these three diffraction/interference patterns, obtained by diffracting laser light through two slits of width a, a distance d apart:



Classify these three patterns in order of a and in order of d.

<u>CLASSIFYING IN ORDER OF a</u>

To classify in order of a, we need to look at the larger envelope. The larger the envelope, the smaller a. So the ordering in this case is



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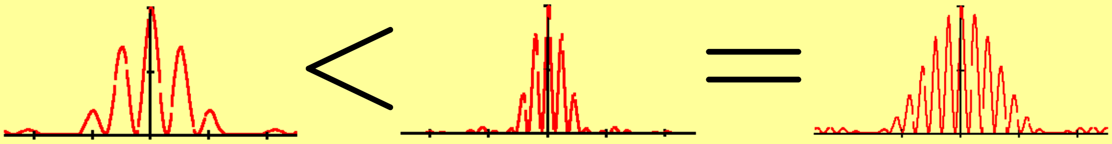
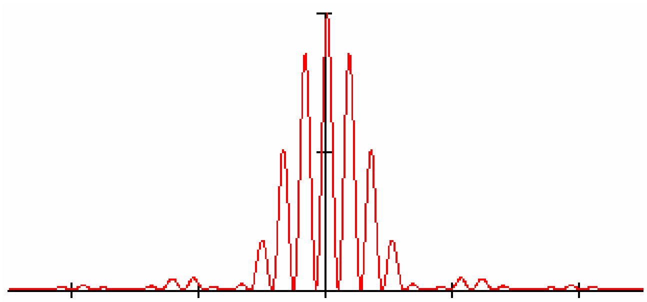
<u>CLASSIFYING IN ORDER OF *d*</u>

To classify in order of d, we need to look at the small "squiggles". The further apart these squiggles are, the <u>smaller</u> d. Therefore, in this case



Here's a fourth pattern, obtained by changing the wavelength of light in *one* of the cases above:



Which of the patterns above was used, and was the wavelength made longer or shorter?

Both the width of the envelope maxima and of the individual squiggle maxima are related to λ. In fact, in both cases, $\Delta y \propto \lambda$. So we expect a change in wavelength to change *both* these features.

We can exclude two of the patterns:

- It isn't the third pattern above, because this new pattern has the same envelope size, and we know that this would have had to change with wavelength.
- It isn't the first pattern, because even though the envelope size has shrunk, the individual squiggle sizes have not.

It must therefore be the second pattern.

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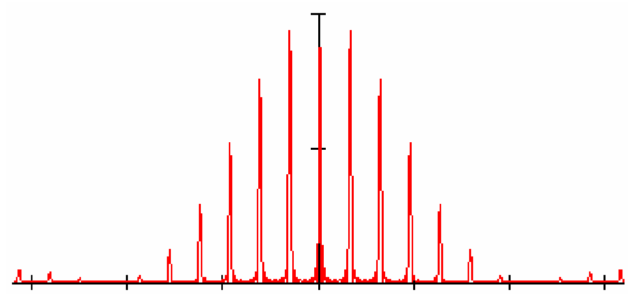
Since both the envelope and the squiggles have shrunk, the wavelength of the light must have got **<u>shorter</u>**.

Finally, the situation that gave rise to the first pattern above was altered in some way, and the resulting diffraction pattern looked like this



How was the situation changed?

The pattern exactly the same, except that the peaks are much sharper. This happens when the number of slits is increased. The pattern above was made with 2 slits, this one was probably made with many more.

Question 5 (*spring 2008 exam*)

The magnetic vector of a plane electromagnetic wave is described as follows

$$\boldsymbol{B} = \left(1\ \mu\text{T}\right)\cos\left[\left(10\ \text{m}^{-1}\right)x + \left(3\cdot 10^{9}\ \text{s}^{-1}\right)t\right]\boldsymbol{j}$$

Which of the following is true?

1. The wave has velocity $\boldsymbol{v} = \left(3\cdot 10^{8}\,\text{ms}^{-1}\right)\boldsymbol{i}$ and angular frequency $\omega = 3\cdot 10^{9}\,\text{s}^{-1}$
2. At $x = 0$ and $t = 0$, the electric field is $\boldsymbol{E} = 150\,\text{V/m}\,\boldsymbol{k}$
3. The intensity (time averaged Poynting vector magnitude) is $I = 1500\,/\,4\pi\ \text{W/m}^2$
4. At $x = 0$, the Poynting vector $\boldsymbol{S} \propto \cos^2\left(\left[3\cdot 10^{9}\,\text{s}^{-1}\right]t\right)\boldsymbol{j}$

The answer is 3. Let's see why the others are wrong

1. In this case, the wavefunction is in terms of $kx + \omega t$. That "plus" sign tells us that the wave is travelling in the $-\boldsymbol{i}$ direction, not the $\boldsymbol{i}$ direction.
2. We know that $E_0 / B_0 = c$, and so in this case, $E_0 = 300\,\text{N/C}$. At $t = 0$ and $x = 0$, the magnitude of $\boldsymbol{E}$ will therefore be 300 N/C, not 150 N/C.
3. This is correct. The magnitude of the Poynting vector is [I have omitted units for clarity – you should check that they match]

   $$\frac{1}{\mu_0}\left(300\times 1\times 10^{-6}\right)\cos^2\left[10x + \left(3\cdot 10^{9}\right)\omega\right]$$

   Averaging $\cos^2$ gives a factor of ½

   $$\frac{150\times 10^{-6}}{\mu_0}$$

   Remembering that $\mu_0 = 4\pi\times 10^{-7}$, we get

   $$\frac{150\times 10^{-6}\times 10^{7}}{4\pi} = \frac{1500}{4\pi}$$
4. The wave is not travelling in the $\boldsymbol{j}$ direction.

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