8.01 IAP Mechanics ReView Course

January 2009

Practice Problems

This document contains a large number of practice problems on the material in 8.01. Practice makes perfect, especially in physics, and doing problems and understanding them is probably the best way for you to get up to scratch on the material in 8.01. We really hope you find these useful. If you find any errors or have any questions, comments, suggestions, personal stories or jokes, please do email me at guetta@mit.edu.

The problems are all provided with short solutions so that you can check your answer and see if you have it correct. Full solutions will *not* be provided, but we will be available throughout the IAP to help you with any problems you're having difficulty with. Check the "instructors" section of the Stellar website (http://tinyurl.com/801rev) for our office hours.

I have given each problem a "difficulty rating". Like all ratings, these are somewhat arbitrary, but they'll hopefully help you get a feel of how tough the problem is. Here's a key to my ratings:

*	Short and/or straightforward question.
**	Slightly more algebraically involved question, but still reasonably straightforward (or involving simpler concepts).
***	Harder question. Perhaps still straightforward, but using much more difficult or tricky concepts.
****	Question that is algebraically involved *and* involving difficult or tricky concepts. There might be a "trick" to solving the question.
*****	Exceptionally difficult question, either because little information has been given, or because a very clever "trick" is required to find the solution.

Questions rated *** or **** are fair play for the exam. Others are probably too easy or too hard. (Note, however, that even some questions labelled *** or **** are too short to be exam questions in their own right).

I have written/selected most of the problems to be of level *** or ****, to give you as much practice at exam level as possible. But if you'd like a few more easier/harder(!) problems, let me know and I can write some.

In answering these questions, you may find the following moments of inertia useful:

- Sphere mass M and radius R about a diameter
$$I = \frac{2MR^2}{5}$$
- Cube mass M and side L about an axis through the centre of one of its faces
$$I = \frac{ML^2}{6}$$
- Disc of mass M and radius R about an axis through its centre and perpendicular to its plane
$$I = \frac{MR^2}{2}$$
- Disc of mass M and radius R about an axis along its diameter
$$I_z = \frac{MR^2}{4}$$
- Hoop of mass M and radius R about an axis through its centre and perpendicular to its plane
$$I = MR^2$$
- Rod of length L and mass M, about a perpendicular axis through its centre.
$$I = \frac{ML^2}{12}$$
- Rod of length L and mass M, about one of its ends.
$$I = \frac{ML^2}{3}$$

<u>Question 1 (*)</u>

A certain bedspring is observed to compress by 1.00 cm when a certain person is at rest atop the spring. What would be the spring's compression if the same person was dropped onto the spring from a height of 0.250 m above the top of the spring? Assume the person is dropped from rest.

Answer: 8.14 cm

Solution:

Presumably, the point of this problem is first to deduce something about the spring using the first piece of information, and then to answer the question. We'll take the (always sensible) approach of expressing our answers in terms of unknowns, and only plugging in numbers at the end. In this case:

- $x = 1$ cm $= 0.01$ m will be the amount the spring compressed when the person was put onto it.
- $H = 0.250$ m will be the height above the spring at which the person was dropped.
- X will be the (unknown) amount the spring compresses when the person is dropped (this is what we need to find)
- m will be the (unknown) mass of the person
- k will be the (unknown) spring constant

First step

Consider the spring at equilibrium. It compresses by a distance x and so exerts a force kx. The weight of the person is mg. Thus

$$mg = kx$$

$$k = \frac{mg}{x}$$

We have therefore deduced something about the spring.

Note: you might have been tempted to use conservation of energy here (potential energy lost by the body = potential energy gained in the spring, but that would have given you the incorrect answer $k = 2mg / x$. The reason is subtle. Using conservation of energy would imply that the body was placed on top of the spring and dropped, and would find the lowest point reached in the subsequent harmonic motion. The case in the question is different, though; it is clearly a situation in which the body is gently lowered until it reaches equilibrium. At that point, therefore, we need to use the balance of forces method above. We could, also, use the energy method and divide the answer by 2 (because the equilibrium point is the center point of the simple harmonic motion, and therefore half its amplitude).

Second step

When the person is dropped onto the spring, the situation is identical, except that the height they drop is now $H + X$ (instead of x) and the compression is X. Thus, conservation of energy gives

$$mg\left(H + X\right) = \tfrac{1}{2}kX^2$$

Feeding in our expression for k:

$$mg\left(H + X\right) = \frac{mg}{2x}X^2$$

Re-arranging to find something meaningful about X:

$$2x\left(H + X\right) = X^2$$
$$X^2 - 2xX - 2xH = 0$$

Solving this quadratic

$$X = \frac{2x \pm \sqrt{4x^2 + 8xH}}{2}$$

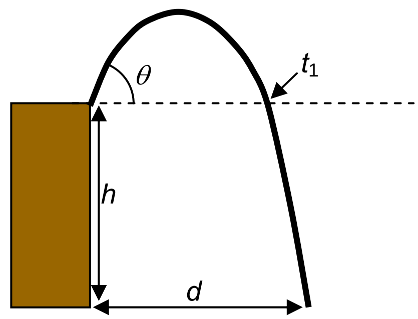
Feeding in our numbers, the two solutions to this quadratic are:

$$X = 8.14 \text{ cm} \qquad \textit{and} \qquad X = -6.14 \text{ cm}$$

Clearly, the second option is unphysical, and the first option is correct, as stated.

However, as good physicists, we should try and understand the second solution – where does it come from? It turns out that our model didn't explicitly include the fact that the spring had to be *compressed*; the equations also allowed for the spring to be stretched. Thus, the second solution corresponds to the spring being stretched by 6.14 cm, which would also balance energy. Of course, it's completely impossible since the body is dropping on the spring, but our equations didn't specify that.

Question 2 (**)



A projectile is launched from a cliff a height h above the ground at an angle θ above the horizontal. After a time t_1 has elapsed since the launch, the projectile passes the level of the cliff top moving downward. It eventually lands on the ground a horizontal distance d from its launch site. Find θ in terms of the other given quantities and the acceleration of gravity (g). (Ignore air resistance).

Answer: $\tan\theta = \dfrac{t_1^2 g\left[1+\sqrt{1+\dfrac{8h}{gt_1^2}}\right]}{4d}$

Solution:

Let t be the total time of flight, and v be the launch speed (ie: the magnitude of the launch velocity). The question gives us three facts. Let's see what we can get from each one of them:

1. "**The projectile lands at a horizontal distance *d* from the original launch site**"

   The original horizontal velocity was $v\cos\theta$. The horizontal acceleration was 0. The horizontal distance traveled was d, and the time it took was t. We can therefore us

$$d = \left(v\cos\theta \times t\right) + \left(\tfrac{1}{2}\times 0 \times t^2\right)$$
$$\boxed{d = vt\cos\theta}$$

2. "**The projectile landed at a vertical distance h from its launch site**"

   In this case, the original velocity was $v\sin\theta$, the vertical acceleration was $-g$ (remember the minus sign, since we've clearly taken upwards as positive for the velocity) and the vertical distance traveled was $-h$. Thus

$$-h = \left(v\sin\theta \times t\right) - \left(\tfrac{1}{2}gt^2\right)$$

$$\boxed{-h = vt\sin\theta - \frac{gt^2}{2}}$$

3. "**The time was t_1 when the projectile passed the top of the cliff**"

   This is precisely the same calculation as point 2, except that the vertical distance traveled is now 0, and the time is t_1. Thus

$$\boxed{vt_1\sin\theta = \frac{gt_1^2}{2}}$$

It's now a question of dealing with these three equations. Let's first eliminate v using the third equation. From the third equation

$$v = \frac{gt_1}{2\sin\theta}$$

And so the first two equations become

$$-h = \frac{gt_1}{2\sin\theta}t\sin\theta - \frac{gt^2}{2} \Rightarrow -h = \frac{gt_1}{2}t - \frac{gt^2}{2}$$

$$d = \frac{gt_1}{2\sin\theta}t\cos\theta \Rightarrow t = \frac{2d\tan\theta}{gt_1}$$

Feeding in t from the second equation into the first:

$$-h = \frac{gt_1}{2}\frac{2d\tan\theta}{gt_1} - \frac{g}{2}\left(\frac{2d\tan\theta}{gt_1}\right)^2$$

$$-h = d\tan\theta - \frac{2d^2\tan^2\theta}{gt_1^2}$$

$$\frac{2d^2}{gt_1^2}\tan^2\theta - d\tan\theta - h$$

This is a quadratic in $\tan\theta$. Using the quadratic equation:

$$\tan\theta = \frac{d \pm \sqrt{d^2 + 4\dfrac{2d^2}{gt_1^2}h}}{4d^2 \,/\, gt_1^2}$$

$$\tan\theta = \frac{gt_1^2\left(d \pm d\sqrt{1+\dfrac{8h}{gt_1^2}}\right)}{4d^2}$$

$$\tan\theta = \frac{gt_1^2\left(1 \pm \sqrt{1+\dfrac{8h}{gt_1^2}}\right)}{4d}$$

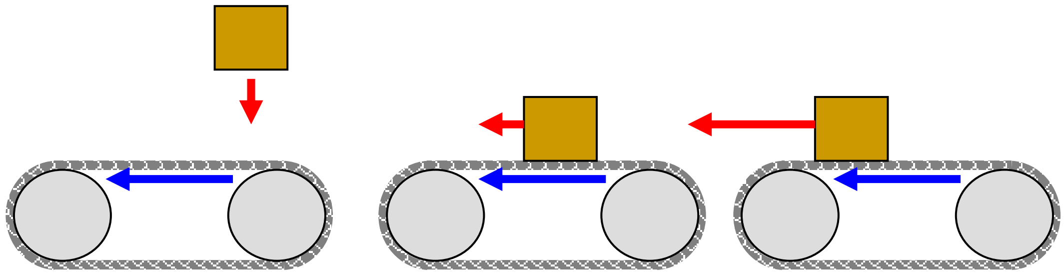
Clearly, the quantity in the square root is greater than 1, and so the negative root makes so sense. Our solution is therefore

$$\boxed{\tan\theta = \frac{gt_1^2\left(1 + \sqrt{1+\dfrac{8h}{gt_1^2}}\right)}{4d}}$$

As advertised.

As usual, it helps to understand where the second root comes from. In this case, it predicts a negative $\tan\theta$ and therefore a negative angle. Looking at point 3 above, this would correspond to a negative $\sin\theta$ and therefore a negative time t_1. This is not possible; we are clearly told that the time t_1 occurs *after* the stone is thrown. But this constraint was not hard-coded into our equations.

Question 3 (****)



A 300 kg crate is dropped vertically onto a conveyor belt that is moving at 1.20 m/s. A motor maintains the belt's constant speed. The belt initially slides under the crate, with a coefficient of friction of 0.400. After a short time, the crate is moving at the speed of the belt. During the period in which the crate is being accelerated, find the work done by the motor which drives the belt.

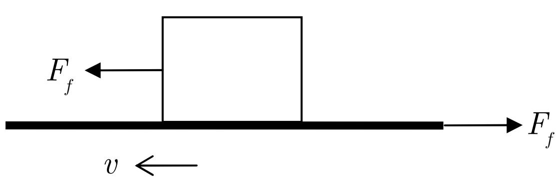
Answer: 432 J

Solution:

This is a fascinating (and rather difficult) problem, which can be dealt with in one of two ways:

In the ground frame

Let's draw a free body diagram for the belt and the crate. Ignoring gravity and the normal force, which cancel each other out, the only forces that remain are the two Newton-pair friction forces – of the belt on the crate, and of the crate on the belt:



$$\boxed{v = 1.2\,\mathrm{m/s}}$$
$$\boxed{F_f = \mu m g = 1177.2}$$

The key insight is to realize that only the belt matters here, because it is the *belt's* motor that we're interested in. Now, let's go through the thought process you'd use to get the answer:

- We need to calculate **Work = Force × Distance** for the belt. We know the force, and so we need the **distance** the belt travels.
- We know how fast the belt is traveling, so if we could find out **how long** the block takes to move with the belt, we can find how fast is traveled.
- The block is being acted on by a constant force, and so we can use kinematics.

Following our plan, we first use kinematics to work out how long the block takes to come to a stop. We know:

- The original velocity of the block = 0 m/s
- The final velocity of the block = v = 1.2 m/s
- The acceleration of the block = $F_f / m = \mu g = 3.924$ m/s^2
- We'd like to find the time…

The appropriate equation is

$$v_f = v_0 + at$$
$$t = \frac{v}{a} = \frac{v}{\mu g}$$

Given this time, we can deduce that the distance the belt traveled was

$$d = vt$$
$$d = \frac{v^2}{\mu g}$$

Using this, we can find the work done by the belt

$$W = F_f d$$
$$W = \mu mg \frac{v^2}{\mu g}$$
$$W = mv^2$$
$$\boxed{W = 432\,\mathrm{J}}$$

In the ground and moving frame

This method is more conceptually difficult, but algebraically much easier. The key insight is to note that the motor is doing two kinds of work here

1. It is accelerating the block, and giving it kinetic energy
2. It is dissipating energy to friction, as the block slips.

We can work out the amount of energy involved in each case

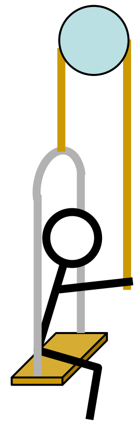
1. In the ground frame, the block starts at rest and eventually moves at speed v. The kinetic energy gained is $\boxed{\frac{1}{2}mv^2}$.
2. By moving to the frame of the conveyor belt, we can find the amount of energy lost to friction. In that frame, the block originally has energy v and then slows down to rest, with all the original energy dissipated to friction. In total, the energy lost to friction is $\boxed{\frac{1}{2}mv^2}$.

The total work down by the belt is then

$$W = \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2 = mv^2$$
$$\boxed{W = 432\,\mathrm{J}}$$

As above.

Question 4 (**)

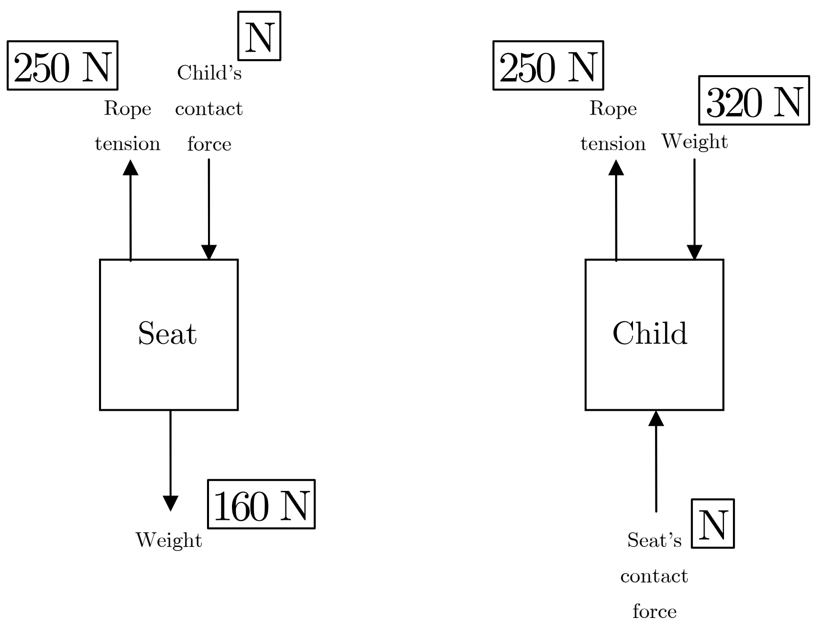


A child is sitting in a chair connected to a rope that passes over a frictionless pulley. The child pulls on the loose end of the rope with a force of 250 N. The child's weight is 320 N and the chair weighs 160 N. The child is accelerating. Find the force that the seat of the chair exerts on the child.

Answer: 83 N

Solution:

Let's draw some free-body diagrams:



(Note that the rope tension is the same in both parts of the rope because the pulley is perfect).

The child always stays in contact with the seat, so the child and seat must be accelerating at the same rate. Thus, the resultant force on each body must be the same. Taking upwards as positive, we get:

$$250 - N - 160 = 250 - 320 + N$$
$$160 = 2N$$
$$\boxed{N = 80 \ \text{N}}$$

Question 5 (**)

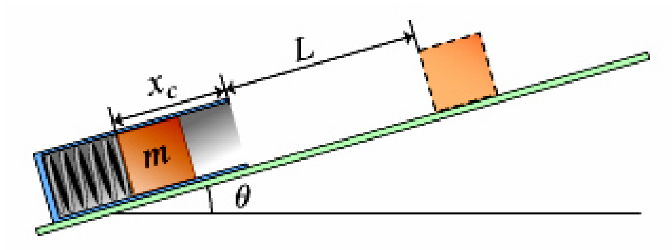
A 5 kg monkey begins climbing up a light rope that passes over a massless, frictionless pulley in an attempt to get to a 5 kg bunch of bananas that hangs from the other side. If the monkey accelerates upward at 1.0 m/s^2, what is the acceleration of the center of mass of the monkey-pulley-bananas system? What external force produces this acceleration?

Answer: 1.0 m/s^2

Solution: Each object only moves under the action of two forces – their (identical) weights and (identical) tensions from the rope. Thus, the acceleration of the two objects must be the same, and the bananas are also accelerating upwards at the same speed (the monkey has little chance of actually getting there!) Thus, the acceleration of the combined system is indeed 1.0 m/s^2

It is the force of the pulley's axle on the pulley that causes this acceleration to occur. The pulley is being pulled downwards, and so the force on its from its axle will be upwards. This causes the entire system to accelerate.

Question 6 (**)



A block of mass m is placed in a frictionless spring gun at the bottom of the incline so that it compresses the spring by an amount x_c. The spring has spring constant k. The incline makes an angle θ with the horizontal and the coefficient of kinetic friction between the block and the incline is μ_k. The block is released, exits the muzzle of the gun, and slides up an incline a total distance L. Find an expression for L in terms of the other quantities described plus the acceleration of gravity.

Answer: $\dfrac{\frac{1}{2}kx_c^2 - mgx_c\sin\theta}{mg\left(\sin\theta + \mu_k\cos\theta\right)}$

Solution:

An energy method seems the most judicious in this case. First, we note that the energy stored in the string originally is given by

$$E_{\text{spring}} = \frac{1}{2}kx_c^2$$

This energy is converted to gravitational potential energy as the block moves up the slope, and into heat as the block gets slowed down by friction. Let's deal with each case:

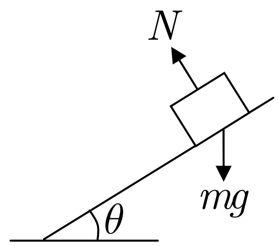
- **Gravitational energy** – if the block moves a distance $L + x_c$ along the slope and gun, we have

$$E_{\text{gravitational}} = mg\left(L + x_c\right)\sin\theta$$

- **Energy dissipated to heat** –the block moves a distance L along the slope (remember – the distance x_c is moved while the block is in the gun, where there is no friction from the slope). Therefore

$$\Delta E_{\text{friction}} = F_f L = \mu_k N L$$

Where F_f is the magnitude of the frictional force, and N is the magnitude of the contact force. To find this magnitude, we draw a free-body diagram for the block:



Resolving perpendicular to the plane (in which direction there is no motion), we obtain

$$N = mg\cos\theta$$

And so

$$\Delta E_{\text{friction}} = \mu_k mg\cos\theta L$$

Equating energy lost with energy gained, we get

$$E_{\text{spring}} = E_{\text{gravitational}} + \Delta E_{\text{friction}}$$

$$\frac{1}{2}kx_c^2 = mg\left(L + x_c\right)\sin\theta + \mu_k mgL\cos\theta$$

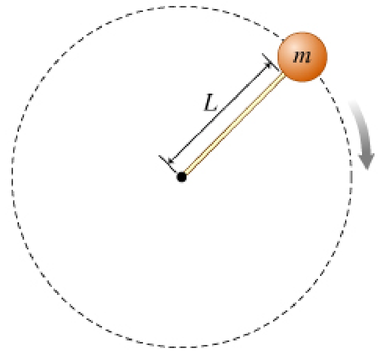
All that remains is to solve for L

$$\left(mg\sin\theta + \mu_k mg\cos\theta\right)L = \frac{1}{2}kx_c^2 - mgx_c\sin\theta$$

$$\boxed{L = \frac{\frac{1}{2}kx_c^2 - mgx_c\sin\theta}{mg\left(\sin\theta + \mu_k\cos\theta\right)}}$$

As advertised.

Question 7 (**)



A ball of mass m is attached to a string of length L. It is being swung in a vertical circle with enough speed so that the string remains taut throughout the ball's motion. Assume that the ball travels freely in this vertical circle with negligible loss of total mechanical energy. At the top and bottom of the vertical circle, the magnitude of the tension in the string is T_{top} and T_{bot}, respectively. Find the difference $T_{\text{bot}} - T_{\text{top}}$.

Answer: $6mg$

Solution:

In all cases, the resultant force towards the centre of the circle has a magnitude

$$F_c = \frac{mv^2}{L}$$

where v is the speed of the ball at that point. This is provided by a combination of the weight of the ball and of the tension in the string. Let v_{top} and v_{bot} be the velocities of the ball at the top and bottom of the circle respectively. Now:

- At the top of the circle, gravity *and* tension are pulling downwards, so

$$F_{c,\text{top}} = \frac{mv_{\text{top}}^2}{L} = T_{\text{top}} + mg$$

- At the bottom of the circle, the tension is pulling inwards, but gravity is pulling outwards, and so

$$F_{c,\text{bot}} = \frac{mv_{\text{bot}}^2}{L} = T_{\text{bot}} - mg$$

So far, therefore, we have

$$\boxed{T_{\text{bot}} - T_{\text{top}} = \frac{m}{L}\left(v_{\text{bot}}^2 - v_{\text{top}}^2\right) + 2mg}$$

Finally, we can use conservation of energy to relate the v_{top} and v_{bot}:

$$\text{KE}_{\text{top}} + \text{PE}_{\text{top}} = \text{KE}_{\text{bot}}$$
$$\frac{1}{2}mv_{\text{top}}^2 + 2mgL = \frac{1}{2}mv_{\text{bot}}^2$$
$$4gL = v_{\text{bot}}^2 - v_{\text{top}}^2$$

Feeding this last result into the equation above, we get

$$T_{\text{bot}} - T_{\text{top}} = 4gL\frac{m}{L} + 2mg$$
$$\boxed{T_{\text{bot}} - T_{\text{top}} = 6mg}$$

As advertised.

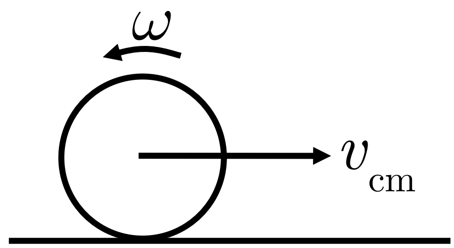
Question 8 (****)

A hot air balloon of mass M is stationary (with respect to the ground) in mid-air. A passenger of mass m climbs out and slides down a rope with constant velocity v with respect to the balloon. With what velocity (magnitude and direction) relative to the ground does the balloon move? What happens if the passenger then stops sliding? Now, answer the following questions:

- Check, mathematically, whether energy is conserved while the passenger is sliding. Does your answer make sense? Shouldn't there be some rubbing between the passenger's hand and the rope?
- It seems that energy is "created" when both person and balloon start moving, and "lost" when both stop. How does that happen? Show, mathematically, that whatever mechanism you come up with "creates" the required energy.

Answer: $\dfrac{m}{m+M}v$ upward.

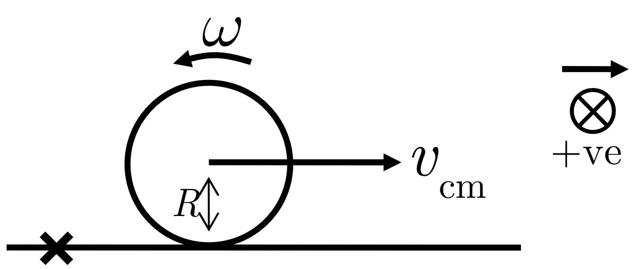
Question 9 (***)



A solid spherical ball of radius R is sliding along a surface and rotating the "wrong" way (see the figure above). There is friction between the ball and the surface. What is the minimum initial value of ω in terms of the initial speed of the ball's center of mass (v_{cm}) in order to ensure that the ball ends up moving to the left in the picture above?

Answer: $\omega \geq 5v_{\text{cm}} / 2R$

Solution: In this case, it seems best to use conservation of angular momentum about a point P on the ground, and consider *into* the page and *to the right* as positive



Consider the angular momentum at the start of the motion, as depicted in the diagram above. Let M be the mass of the ball:

$$L_{\text{initial}} = L_{\text{translational}} + L_{\text{rotational}}$$
$$L_{\text{initial}} = MRv_{\text{cm}} - I\omega$$

(Note **very carefully** the sign of each term in the equation above, and understand what's happening).

Let the final velocity of the centre of mass of the ball be $v_{\text{cm},f}$ to the right and let the final angular velocity of the ball be ω_f ***clockwise***. The final angular momentum is then

$$L_{\text{final}} = MRv_{\text{cm},f} + I\omega_f$$

Angular momentum is conserved, and so

$$MRv_{\text{cm},f} + I\omega_f = MRv_{\text{cm}} - I\omega$$

$$v_{\text{cm},f} = \frac{MRv_{\text{cm}} - I\omega - I\omega_f}{MR}$$

We further assumed that the ball was moving to the right and rotating clockwise, and so $v_{\text{cm},f} = \omega_f R$. Feeding this into the equation, we obtain

$$v_{\text{cm},f} = \frac{MRv_{\text{cm}} - I\omega - (Iv_{\text{cm},f} / R)}{MR}$$

Re-arranging

$$\left(MR + \frac{I}{R}\right)v_{\text{cm},f} = MRv_{\text{cm}} - I\omega$$

$$\left(\frac{MR^2 + I}{R}\right)v_{\text{cm},f} = MRv_{\text{cm}} - I\omega$$

$$v_{\text{cm},f} = R\frac{MRv_{\text{cm}} - I\omega}{MR^2 + I}$$

We want the ball to move to the **left**, and so we want $v_{\text{cm},f} \leq 0$. As such

$$R\frac{MRv_{\text{cm}} - I\omega}{MR^2 + I} \leq 0$$

$$MRv_{\text{cm}} - I\omega \leq 0$$

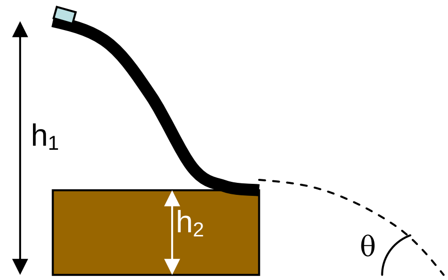
$$\omega \geq \frac{MRv_{\text{cm}}}{I}$$

From the formula sheet, we find that for a sphere, $I = 2MR^2 / 5$, and so

$$\omega \geq \frac{5MRv_{\text{cm}}}{2MR^2}$$

$$\boxed{\omega \geq \frac{5v_{\text{cm}}}{2R}}$$

Question 10 (***)



A block starts from rest atop a frictionless slide at a height h_1 above the ground. The block leaves the slide moving perfectly horizontally at a height h_2 above the ground. The block eventually hits the ground traveling at an angle θ below the horizontal. Find h_2 in terms of the other given quantities and the acceleration of gravity (g).

Answer: $h_2 = h_1 \sin^2\theta$

Solution:

The first step is to work out how fast the block is traveling when it leaves the slide. This can be done by conservation of energy:

$$\tfrac{1}{2}mv^2 = mg\left(h_1 - h_2\right)$$

$$\boxed{v = \sqrt{2g\left(h_1 - h_2\right)}}$$

The second step is the projectile motion of the block in the air. The first thing you should ask yourself is how the angle θ enters into the picture. The answer is that it links the final velocity in the x direction to that in the y direction. In fact, we have that

$$v_{f,x} = v_f \cos\theta \text{ and } v_{f,y} = v_f \sin\theta$$

Dividing one by the other, we obtain

$$\boxed{\frac{v_{f,y}}{v_{f,x}} = \tan\theta}$$

The next thing to note is that after it has left the slide, the motion of the block is simply projectile motion with $v_{0,x} = \sqrt{2g(h_1 - h_2)}$ and $v_{0,y} = 0$.

We already have an expression relating the final horizontal and vertical velocities to the angle, so clearly, what we want here is an expression for the final velocities of this block as it hits the ground. The horizontal component is easy

$$v_{f,x} = v_{0,x} = \sqrt{2g\left(h_1 - h_2\right)}$$

The vertical component is a bit more tricky. We *could* involve the time t, but that would introduce an extra variable. A better choice is to say

$$v_{f,y}^2 = v_{0,y}^2 + 2gx$$
$$v_{f,y} = \sqrt{2gh_2}$$

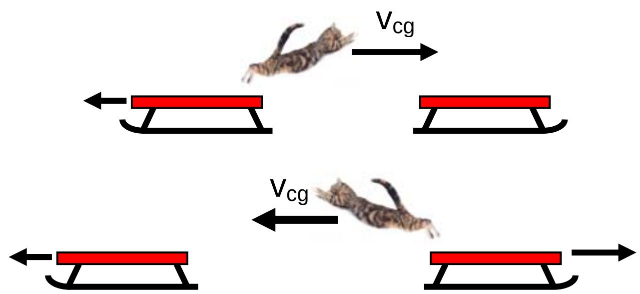
Putting these two together, we then get

$$\frac{v_{f,y}}{v_{f,x}} = \frac{\sqrt{2gh_2}}{\sqrt{2g\left(h_1 - h_2\right)}} = \sqrt{\frac{h_2}{h_1 - h_2}}$$

Using the equation in a box above:

$$\sqrt{\frac{h_2}{h_1 - h_2}} = \tan\theta$$
$$\frac{h_2}{h_1 - h_2} = \tan^2\theta$$
$$\frac{h_1 - h_2}{h_2} = \cot^2\theta$$
$$\frac{h_1}{h_2} - 1 = \frac{1}{\sin^2\theta} - 1$$
$$\boxed{h_2 = h_1 \sin^2\theta}$$

Question 11 (***)

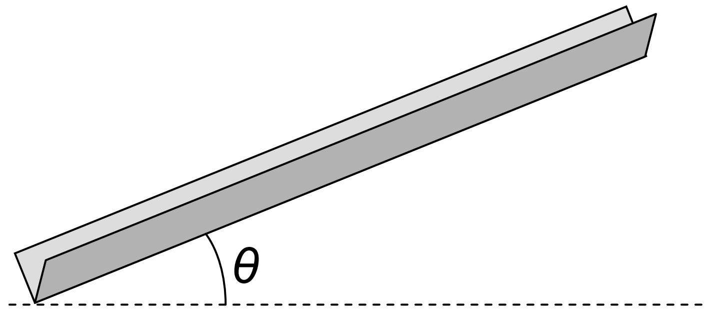


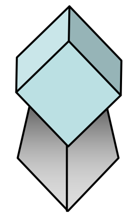
A strange cat with a mass m_c is sitting at rest on the left sled of a pair of identical sleds. The sleds have mass m_s and sit on frictionless ice. Suddenly, the cat leaps to the right sled, traveling with a horizontal speed v_{cg} measured with respect to the ground. The instant the cat reaches the right sled, it turns around and leaps back to the left sled. The horizontal component of the cat's speed is again v_{cg} measured with respect to the ground. What is the final speed of each sled in terms of the masses of the cat and sleds and the cat's leaping speed? (The cat remains on the left sled after its return).

Answer: $v_{\text{left sled}} = \dfrac{2m_c v_{cg}}{m_s + m_c}$ and $v_{\text{right sled}} = \dfrac{2m_c v_{cg}}{m_s}$

Solution: Let m_c be the mass of the cat, and m_s be the mass of the sled. Since the ice is assumed to be frictionless, there is no external force acting on the cat-sled system, and so momentum is conserved at each "jump".

Question 12 (***)

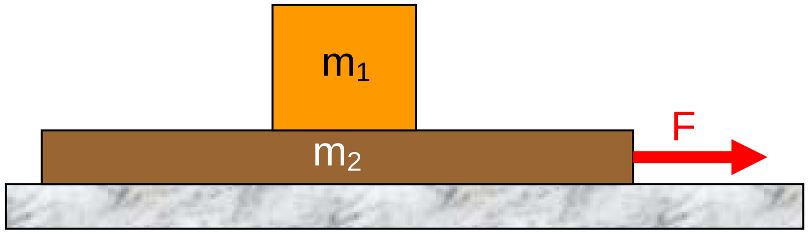




A cube of mass m slides down an inclined right-angle trough. If the coefficient of friction between the cube and the trough is μ, find the acceleration of the block.

Answer: $g\left(\sin\theta - \mu_k \sqrt{2}\cos\theta\right)$

Question 13 (***)



A small box of mass m_1 is sitting on a board of mass m_2 and length L. The board rests on a frictionless horizontal surface. The coefficient of static friction between the board and the box is μ_s. The coefficient of kinetic friction between the board and the box is μ_k where $\mu_k < \mu_s$. Find $F_{\min}$, the constant force with the least magnitude that must be applied to the board in order to pull the board out from under the box (which will then fall off of the opposite end of the board).

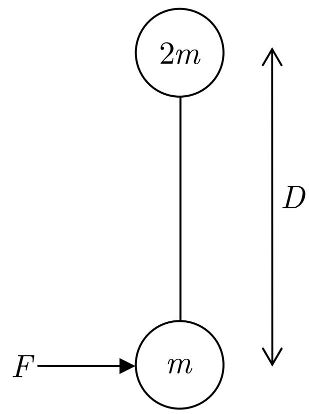
Answer: $F \geq \mu_s \left(m_1 + m_2\right) g$

Question 14 (***)

An object of mass m is traveling on a horizontal surface. There is a coefficient of kinetic friction μ_k between the object and the surface. The object has speed v when it reaches $x = 0$ and later encounters a spring. The object compresses the spring, stops, and then recoils and travels in the opposite direction. When the object reaches $x = 0$ on its return trip, it stops. From this information, find an expression for the spring constant, k.

Answer: $k = \dfrac{8\mu^2 mg^2}{v^2}$

Question 15 (***)

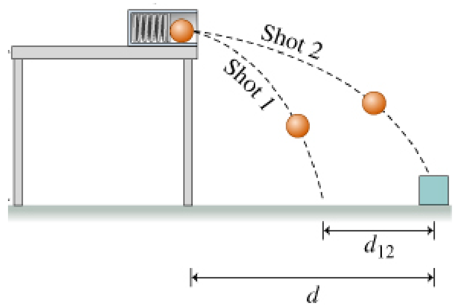


Two small disks of masses $2m$ and m are connected by a rod of negligible mass of length D, forming a rigid body. The body is placed on a horizontal and frictionless table. The rod is initially placed along the vertical as shown in the figure. At time $t = 0$, a force of magnitude F along the positive x-direction is applied at the object of mass m for a short time interval Δt.

What is the angular velocity and the velocity of the center of mass of the rigid body after the force F is turned off?

Answer: $\omega = \dfrac{F\Delta t}{mD}$ and $v_{cm} = \dfrac{F\Delta t}{3m}$

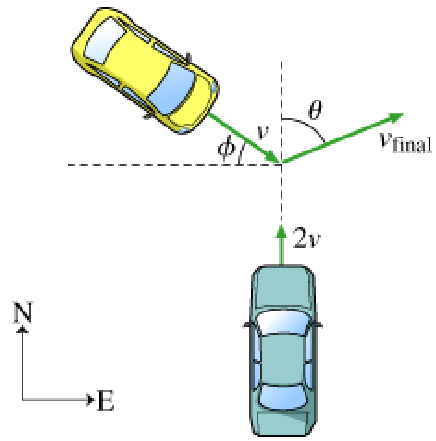
<u>Question 16 (***)</u>



Two children are trying to shoot a marble into a small box using a spring-loaded gun that is fixed on a table and shoots horizontally from the edge of the table. The center of the box is a horizontal distance d from the edge of the table. The first child compresses the spring a distance x_1 and finds that the marble falls short of its target by a horizontal distance d_{12}. By what distance x_2 should the second child compress the spring to score a direct hit on the box?

Answer: $\dfrac{dx_1}{d - d_{12}}$

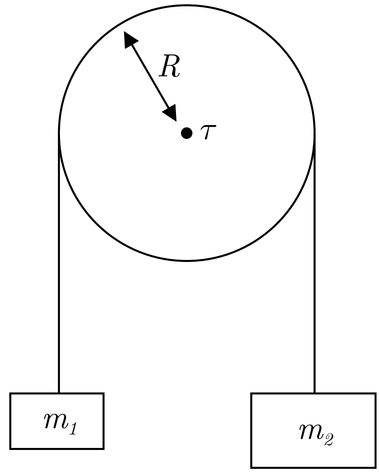
Question 17 (***)



Two cars, both of mass m, collide and stick together. Prior to the collision, one car had been traveling north at speed $2v$, while the second was traveling at speed v at an angle φ south of east (as indicated in the figure). Find the magnitude and direction of the velocity of the two-car system immediately after the collision.

Answer: $v_{\text{final}} = \dfrac{v}{2}\sqrt{5 - 4\sin\phi}$ and $\tan\theta = \dfrac{\cos\phi}{2 - \sin\phi}$

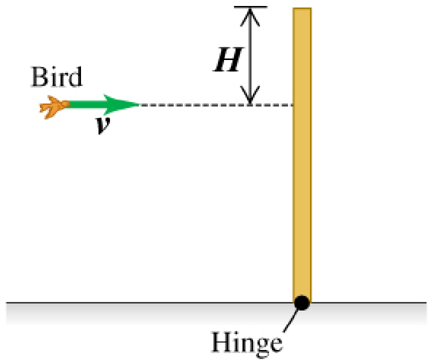
Question 18 (***)



A block of mass m_1 is attached to a light, inextensible string which passes over a pulley of mass M and radius R which is free to rotate about a fixed bearing and is then attached to another block of mass $m_2 > m_1$. The bearing about which the pulley is attached exerts a torque $\tau = \beta R$ on the pulley, against its direction of rotation. The system is released from rest at $t = 0$. **(A)** Assuming that β is constant, find acceleration of mass m_1, in terms of τ. **(B)** Assuming that the frictional torque is time dependent, so that $\beta = 2t$, how long will the block take to come to instantaneous rest?

Answer: $a_1 = \dfrac{(m_2 - m_1)g - \beta}{\frac{1}{2}M + m_2 + m_1}$ and it will come to rest after a time $t = \left(m_2 - m_1\right)g$

Question 19 (***)



A bird of mass m is flying horizontally at speed v, not paying much attention, when it suddenly flies into a stationary vertical bar, hitting it a distance H below the top. The bar is uniform, with mass M and length d, and is hinged at its base. The collision stuns the bird so that it just drops to the ground afterward (but soon recovers to fly happily away). What is the angular velocity of the bar just before it hits the ground?

Answer: $\omega = \sqrt{\left(\dfrac{3mv(d-H)}{Md^2}\right)^2 + \dfrac{3g}{d}}$

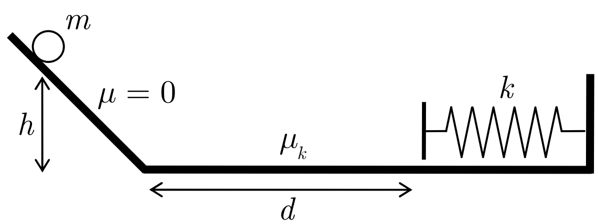
Question 20 (***)

A ball of mass m and radius a resting on a horizontal surface is set rolling without slipping at a speed u by a single horizontal impulse. How large is the impulse, and where is it applied?

Answers: Impulse is mu, and it is applied $2a/5$ above the centre of mass.

Question 21 (***)

A point particle of mass m slides a vertical distance h down a frictionless inclined plane. It then travels a distance d along a plane with which it has coefficient of friction μ_k before colliding elastically with a massless spring of spring constant k. The particle compresses the spring a distance x before coming to rest.



Find the height h in terms of the other variables in the problem and g.

Answer: $h = \mu_k(d + x) + \frac{kx^2}{2mg}$

Question 22 (***)

A uniform rod of length l and mass M rests on a smooth horizontal table. A point mass $m = \frac{1}{2}M$ moving at right angles to the rod with speed v collides with one end of the rod and sticks to it.

(A) Describe the subsequent motion of the rod

(B) Derive the velocity (speed and direction) of the centre of mass (of the combined rod and mass) after the collision.

(C) Derive the angular speed of the combined rod and mass after the collision.

(D) Find the point on the rod that is stationary immediately after the collision.

Answers:

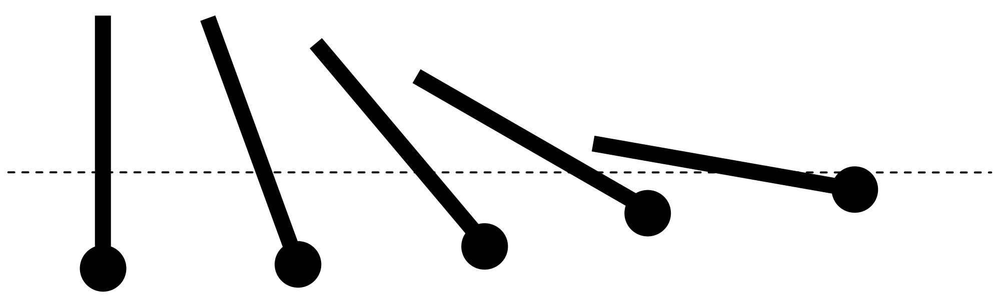
(B) $v/3$

(C) v/l

(D) The point $2l/3$ from the mass.

Solution:

After the collision, the rod will translate and rotate about its new centre of mass. We'll assume the mass sticks to the bottom of the rod, for simplicity.



[Notice how the centre of mass does not move vertically, since it starts at rest (the mass is not moving horizontally) and there are no external vertical forces present]

No external forces act on the mass-rod system, so the linear velocity of the centre of mass after the collision is easily found using conservation of linear momentum:

$$mv = (m + M)V$$
$$\tfrac{1}{2}Mv = (\tfrac{1}{2}M + M)V$$
$$\boxed{V = v\,/\,3}$$

No external torques act on the mass-rod system (about any point), and so angular momentum is conserved. We'll choose a stationary point that is located at the centre of mass of the mass-rod system (ie: the point *l/3* above the bottom of the rod). For that point, conservation of angular momentum gives

$$mv\frac{l}{3} = I\omega$$
$$\omega = \frac{lMv}{6I}$$

To find I, the combined moment of inertia of the rod-mass system after the collision about our chosen point, we note that it is given by

$$\begin{aligned} I = {} & \text{Moment of inertia of rod about that point} + \\ & \qquad \text{Moment of inertia of particle about that point} \end{aligned}$$

The first part of this equation can be calculated using the parallel axis theorem and realising that our chosen point is at a distance $l/6$ below the centre of mass. The second part can be calculated by noticing that the mass is at a distance $l/3$ from our point. We then get

$$\begin{aligned} 6I &= 6\left(\frac{Ml^2}{12} + M\left[\frac{l}{6}\right]^2\right) + 6m\left(\frac{l}{3}\right)^2 \\ &= \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{3}\right)Ml^2 \\ &= Ml^2 \end{aligned}$$

And so we get

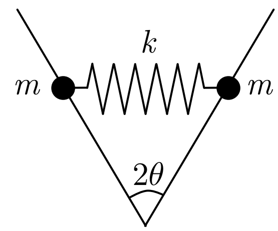
$$\omega = \frac{lMv}{Ml^2}$$
$$\boxed{\omega = v\,/\,l}$$

After the collision, the point at which the forward motion exactly cancels the backwards rotational motion will be stationary. In other words, the point at which

$$\omega R = \frac{v}{3}$$
$$\frac{v}{l}R = \frac{v}{3}$$
$$\boxed{R = \frac{l}{3}}$$

In other words, the point $l/3$ from the centre of mass, or $2l/3$ from the bottom of the rod where the mass struck.

Question 23 (***)

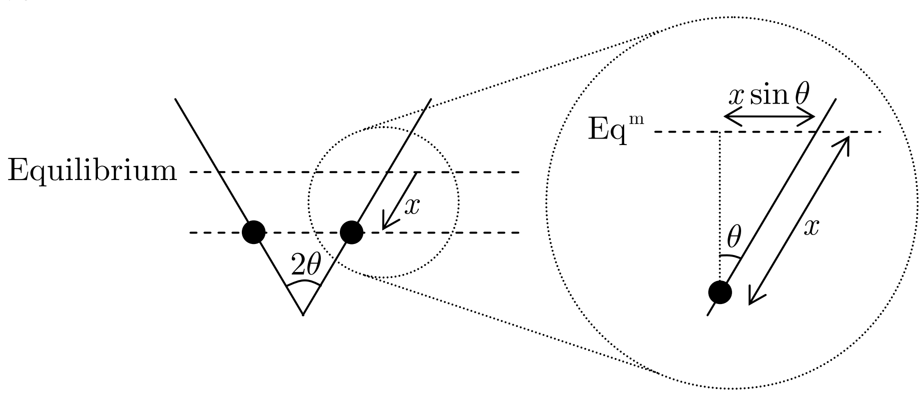


Two particles of mass m are constrained to move along two horizontal frictionless rails that make an angle 2θ with respect to each other. They are connected by a spring with spring constant k. What is the frequency of oscillations for the motion where the two masses always stay parallel to each other (ie: the distance between the meeting point of the rails and each particle is equal)?

Answer: $\omega = \sqrt{\dfrac{2k}{m}}\sin\theta$

Solution:

Let x be the distance that each mass has moved along the rails, with downwards as positive



It is clear from the "zoomed in" diagram that this motion will cause the spring to compress by an amount $2x\sin\theta$ (the "2" is there because it will happen twice – once on each side). This will result in a spring force of $2kx\sin\theta$. The component of this force parallel to the rails is given by $2kx\sin^2\theta$.

We can now find an equation of motion for a single particle, parallel to the rails:

$$ma = F$$

$$ma = -2kx\sin^2\theta$$

$$a = -\frac{2k\sin^2\theta}{m}x$$

(Note the minus sign, because the force tends to *restore* x to its equilibrium position).

We can, however, re-write this as

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{2k\sin^2\theta}{m}x$$

This is the standard form of an SHM equation, with

$$\boxed{\omega = \sqrt{\frac{2k}{m}}\sin\theta}$$

[Note: we have not taken gravity into account, but it makes no difference whatsoever – the frequency remains the same. We leave it as an exercise for the reader to prove this].

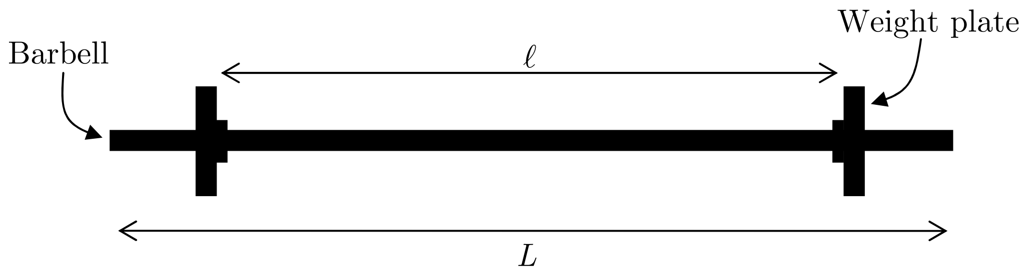
Question 24 (***)

A uniform rod of length L and mass M which is free to rotate on a pivot at its top end is hanging vertically at rest. A bullet of mass m which is travelling horizontally at speed v strikes the lower end of the rod and is brought to rest, so falling to the ground. Find the initial angular speed of the rod.

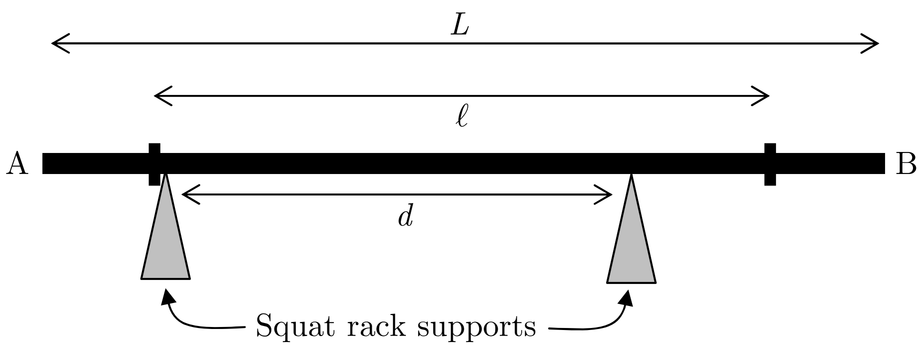
Answer: $\omega = \dfrac{3mv}{ML}$

Question 25 (***)

A barbell is a metal bar on which weights can be loaded for weightlifting. The bar itself has a length L and a mass M, and weights are loaded symmetrically at the ends of the bar, a distance ℓ apart:



A squat rack is a structure that contains two supports, a distance d apart, with $d < \ell$, onto which the barbell can be placed. A member of the MIT football team rests such a barbell (with *no* weights loaded) on a squat rack as follows:



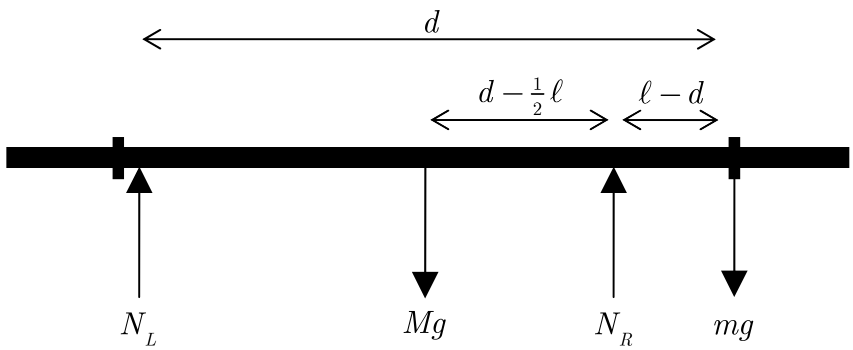
The player needs to load both sides of the bar with the same amount of weight, but is too lazy to do it progressively. Having taken 8.01, he finds an expression for the maximum amount of weight he can load on end B while ensuring the bar does not tip over. Find that expression. Is it dimensionally consistent? Does it work in extreme cases?

For the barbells and racks in the Z-Centre, $L = 2.20\text{m}$, $\ell = 1.42\,\text{m}$, $M = 44.08\text{lb}$ and $d = 1.09\text{m}$. I'll try next time I go to the gym, but I suspect that the formula will slightly overestimate the maximum amount of weight I can load safely. Can you think why? (Hint: it takes more than one weight plate).

Answer: $m = M(d - \frac{1}{2}\ell)\,/\,(\ell - d)$

Solution:

Let's draw a free-body diagram for the barbell with a weight m loaded on one side:



The hardest part of this problem is working out the various distances, so make sure you understand how I did them.

For the barbell to remain static, torques about any point must sum to 0. We choose the point where N_R acts, and we get

$$Mg\left(d-\tfrac{1}{2}\ell\right)=mg\left(\ell-d\right)+N_L d$$

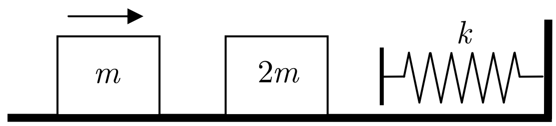
If the weight m is *just* large enough to tip the barbell, N_L will be equal to 0 (because that side of the barbell will *just* be lifted off the support). As such

$$Mg\left(d-\tfrac{1}{2}\ell\right)=mg\left(\ell-d\right)$$

$$\boxed{m=\frac{M\left(d-\frac{1}{2}\ell\right)}{\ell-d}}$$

Question 26 (***)

A block of mass m is moving on a smooth table when it collides elastically with a second block of mass $2m$, which then elastically strikes a massless spring which compresses an amount d before the block comes to rest



Find the original velocity at which the first block was moving, in terms of the other variables in the problem.

Answer: $v = \frac{3}{2} d\sqrt{k / 2m}$

Solution:

There are two stages to this problem

1. The collision
2. The spring compressing

During the collision, both energy and momentum are conserved (because the collision is elastic). Therefore

$$\begin{aligned} mv_{m,\text{before}} &= mv_{m,\text{after}} + 2mv_{2m,\text{after}} \\ \frac{1}{2}mv_{m,\text{before}}^2 &= \frac{1}{2}mv_{m,\text{after}}^2 + mv_{2m,\text{after}}^2 \end{aligned}$$

Simplifying and re-arranging

$$\begin{aligned} v_{m,\text{before}} - 2v_{2m,\text{after}} &= v_{m,\text{after}} \\ v_{m,\text{before}}^2 &= v_{m,\text{after}}^2 + 2v_{2m,\text{after}}^2 \end{aligned}$$

Feeding the second equation into the first, and solving

$$\begin{aligned} v_{m,\text{before}}^2 &= v_{m,\text{before}}^2 + 4v_{2m,\text{after}}^2 - 4v_{m,\text{before}}v_{2m,\text{after}} + 2v_{2m,\text{after}}^2 \\ 6v_{2m,\text{after}}^2 &= 4v_{m,\text{before}}v_{2m,\text{after}} \\ &\boxed{v_{2m,\text{after}} = \frac{2}{3}v_{m,\text{before}}} \end{aligned}$$

In the second stage, energy is conserved as the kinetic energy of the block is transferred to potential energy in the spring:

$$\frac{1}{2}2mv_{2m,\text{after}}^2 = \frac{1}{2}kd^2$$
$$\frac{1}{2}2m\frac{4}{9}v_{m,\text{before}}^2 = \frac{1}{2}kd^2$$
$$v_{m,\text{before}}^2 = \frac{9kd^2}{8m}$$
$$\boxed{v_{m,\text{before}} = \frac{3}{2}d\sqrt{\frac{k}{2m}}}$$

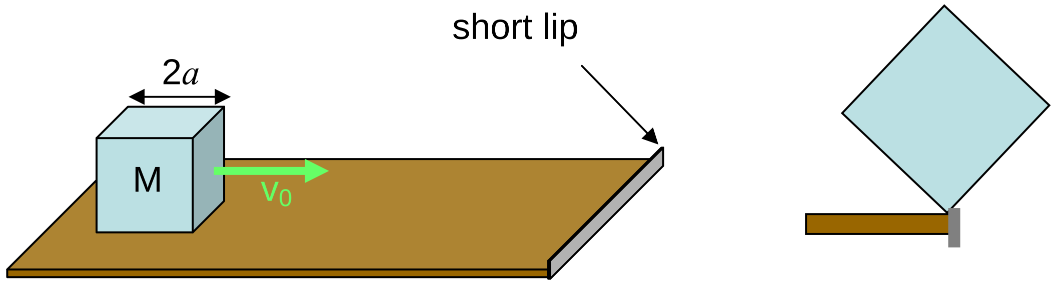
Question 27 (***)

Comet Encke was discovered in 1786 by Pierre Mechain, and in 1822 Johann Encke determined that its period was 3.3 years. It was photographed in 1913 by the telescope at Mt. Wilson at its aphelion (furthest point from sun in the orbit), when it was at a distance $r_a = 6.1 \times 10^{11}\ \text{m}$ from the sun. At its perihelion (closest point from sun in the orbit), its distance from the sun was $r_p = 5.1 \times 10^{10}\ \text{m}$. The universal gravitation constant is $G = 6.67 \times 10^{-11}\ \text{Nm}^2\text{kg}^{-2}$ and the mass of the sun is $m_{\text{sun}} = 2.0 \times 10^{30}\ \text{kg}$.

Write down expressions for the conservation of energy and angular momentum at the aphelion and perihelion, and hence find the speed v_{a} and v_{p} of the comet at its aphelion and perihelion.

Answer: $v_p = 6.94 \times 10^4\ \text{m/s}$ and $v_a = 5.81 \times 10^3\ \text{m/s}$

Question 28 (****)



A cube with sides of length $2a$ and a mass M is moving with an initial speed v_0 along a frictionless table. When the cube reaches the end of the table it is caught abruptly by a short lip and begins to rotate. What is the minimum speed v_0 such that the cube falls off the table?

Answer: $v_0 \geq \sqrt{\frac{16}{3}ag\left(\sqrt{2}-1\right)}$

Solution:

There are three stages in the motion:

1. The block travels along the frictionless table
2. The block hits the small lip
3. The block rotates around the small lip.

First stage

The first stage is trivial, and changes nothing.

Second stage

In the second stage, let's see which conservation laws apply:

- Energy is clearly not conserved – there might be a loud bang during the collision.
- Linear momentum is also not conserved – there is an external force on the block from the lip.

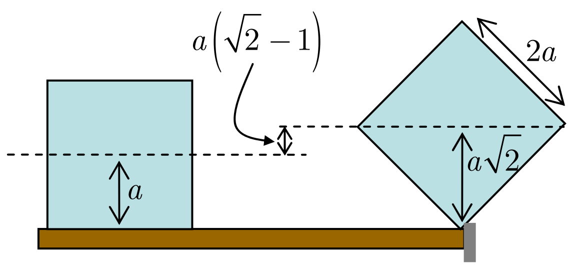
- Angular momentum around the lip, however, *is* conserved, because the only external force in our system acts at the lip.

Thus, if I is the moment of inertia of our cube about its lower right corner, we have that

$$\text{Initial angular momentum} = \text{Final angular momentum}$$
$$Mav_0 = I\omega$$
$$\boxed{\omega = \frac{Mav_0}{I}}$$

Third stage

In the third stage, energy is conserved. For the block to tip over the lip, its centre of mass must end up a distance $a\left(\sqrt{2}-1\right)$ above its original position



In other words, the energy of the rotational motion we calculated in the second stage must be large enough to raise it that amount. In other words, we need

$$\frac{1}{2}I\omega^2 \geq Mga\left(\sqrt{2}-1\right)$$
$$\frac{1}{2}I\left(\frac{M^2a^2v_0^2}{I^2}\right) \geq Mga\left(\sqrt{2}-1\right)$$
$$v_0 \geq \sqrt{\frac{2Ig\left(\sqrt{2}-1\right)}{Ma}}$$

From the formula sheet, we find that the moment of inertia of the cube about the *centre* of one of its faces is

$$I_{\text{cm}} = \frac{2Ma^2}{3}$$

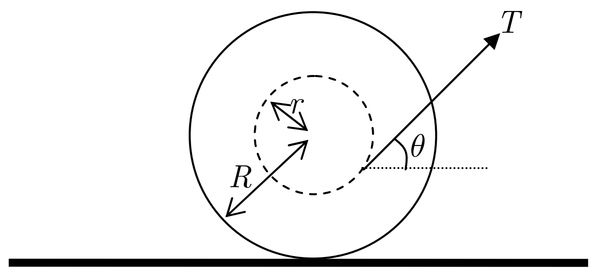
Using the parallel axis theorem, we find that

$$I = \frac{2Ma^2}{3} + M\left(a\sqrt{2}\right)^2$$
$$I = \frac{2Ma^2}{3} + 2Ma^2$$
$$I = \frac{8Ma^2}{3}$$

Feeding this into our equation for v_0 above, we find that

$$v_0 \geq \sqrt{\frac{2 \times 8Ma^2 g\left(\sqrt{2}-1\right)}{3Ma}}$$
$$\boxed{v_0 \geq \sqrt{\frac{16}{3} ag\left(\sqrt{2}-1\right)}}$$

Question 29 (****)



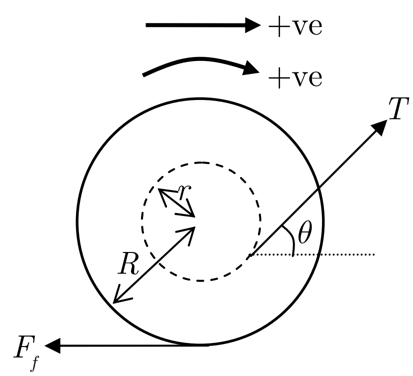
A yo-yo consists of two solid disks, each of mass M and radius R, connected by a central spindle of radius r and negligible mass. A light string is coiled around the central spindle. The yoyo is placed upright on a rough flat surface and the string is pulled gently with a tension T at an angle θ to the horizontal. The pull is gentle enough to ensure that that yo-yo does not slip and that the yo-yo is not lifted off the ground.

What is the acceleration of the centre of mass of the yo-yo?

Answer: $a_{cm} = T\left(\dfrac{\cos\theta - \frac{r}{R}}{3M}\right)$

Solution:

We must consider both the rotational and linear motions. A free-body diagram with sign conventions is essential:



A few points:

- The question tells us there is no vertical motion, and so the weight and normal force are irrelevant. We aren't even given a coefficient of friction, and so the normal force would be useless.
- The friction points the way it does because if the disc *were* to slip (think of giving the string a sharp thug), it would move to the right. And so friction opposes that.
- Taking torques about the centre of mass is our only option, because if we took torques about any stationary point above or below the centre of mass, the exact position at which the string winds around the smaller spindle would matter – information which we do not have at hand.

We can now deal with each part of the motion

Linear motion

As we remarked above, the linear motion will only be horizontal. The equation motion is

$$F = ma$$

$$\boxed{T\cos\theta - F_f = 2Ma}$$

(Note that the mass of the whole yoyo is $2M$ – one M for each of the large discs)

Rotational motion

The rotational equation of motion is

$$\tau = I\alpha$$

$$RF_f - rT = I\alpha$$

$$RF_f - rT = \left(\frac{MR^2}{2} + \frac{MR^2}{2}\right)\alpha$$

$$\boxed{RF_f - rT = MR^2\alpha}$$

Linking equation

The yoyo is rolling without slipping, and so

$$\boxed{a = R\alpha}$$

Convince yourself that the signs are correct – look at the sign convention above and think!

Putting it all together

We need to put everything together to eliminate α and F_f to get a, as required. First, let's feed the first equation into the second to eliminate F_f:

$$R\left(T\cos\theta - 2Ma\right) - rT = MR^2\alpha$$

Next, let's use the last equation to eliminate α:

$$R\left(T\cos\theta - 2Ma\right) - rT = MRa$$

Finally, let's fiddle with it to get a

$$RT\cos\theta - 2MRa - rT = MRa$$

$$3MRa = RT\cos\theta - rT$$

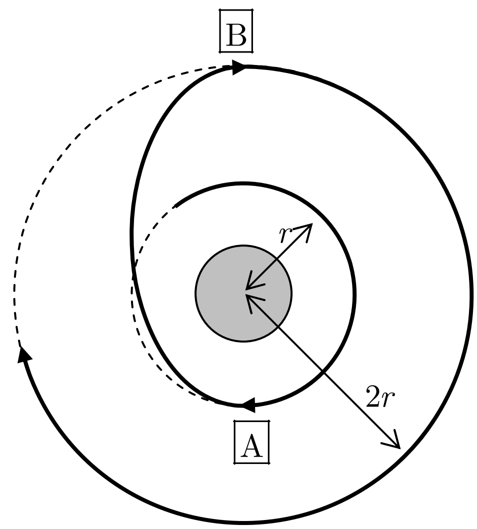
$$a = T\frac{R\cos\theta - r}{3MR}$$

$$\boxed{a = T\left(\frac{\cos\theta - \frac{r}{R}}{3M}\right)}$$

Question 30 (****)

The Hohmann Transfer Orbit maneuver is used by spaceships to transfer from one circular orbit around a planet to another, by temporarily entering an intermediate elliptical orbit.

The spaceship starts on a small circular orbit of radius r around a planet of mass M. At point **A** in the diagram below, it fires its propellers to put itself onto an intermediate elliptical orbit. At point **B**, it fires its propellers again, and ends on the larger circular orbit of radius $2r$:

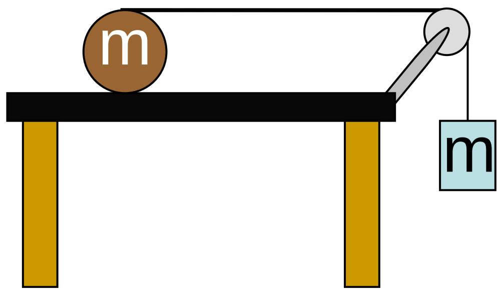


Find Δv_A and Δv_B (the amount by which the velocity needs to be changed at point A and at point B) needed to achieve this travel path. In each case, does the spaceship need to speed up or slow down? You may assume that the propellers are so powerful that the changes in velocity occur instantaneously at each point.

Hint: First find the speed of the spaceship on each of the circular orbits. Then, separately find the speed of the spaceship at each point of the elliptical orbit (if you get *really* stuck, try question 27 first).

Answers: Needs to speed up by $\Delta v_A = \sqrt{\frac{GM}{r}}\left(\sqrt{\frac{4}{3}} - 1\right)$, and then speed up again by $\Delta v_B = \sqrt{\frac{GM}{r}}\left(\sqrt{\frac{1}{2}} - \sqrt{\frac{1}{3}}\right)$.

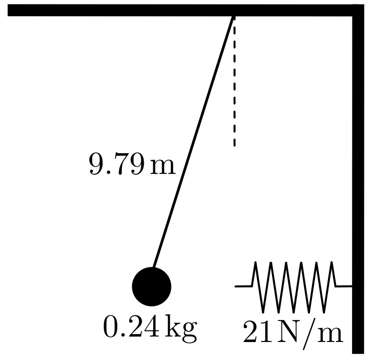
Question 31 (****)



A string is wound around a heavy cylinder of mass m which rests on a table. The string then passes over a massless, frictionless pulley and is attached to a heavy object of mass m which hangs over the side of the table. What will be the acceleration of the hanging object, assuming that the cylinder rolls without slipping?

Answer: 5.6 m/s^2

Question 32 (****)



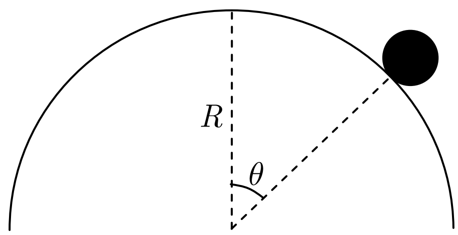
In the figure, a 0.24 kg ball is suspended from a string 9.79 m long and is pulled slightly to the left. As the ball swings through the lowest point of its motion, it encounters a spring of spring constant 21 N/m attached to a wall. The spring pushes against the ball and eventually the ball is returned to its original starting position. Find the period of the oscillation. You may assume that the spring is so strong that the ball hardly moves at all once it has collided with the spring.

Answer: 3.4 s

Question 33 (****)

A ball of mass m and radius a rolls down the surface of a larger fixed rough sphere of radius R, starting from rest at the top.

(A) At the point depicted in the diagram below, when the line between the two centres makes an angle θ to the upwards vertical, what is the speed of the centre of mass of the smaller sphere?



(B) Find the normal force of the larger sphere on the smaller sphere at this point, and **(C)** find the angle at which the smaller sphere leaves the surface of the larger sphere.

Answers:

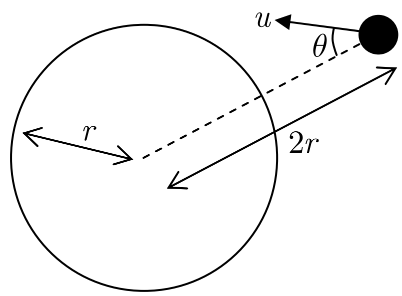
(A) $v = \sqrt{\frac{10}{7} g(R+a)(1-\cos\theta)}$

(B) $N = \frac{1}{7} mg\left(17\cos\theta - 10\right)$

(C) Leaves larger sphere when $\cos\theta = 10 / 17$

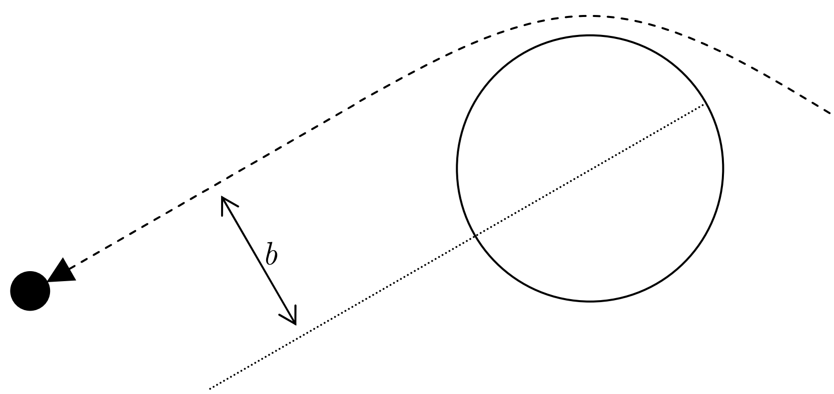
Question 34 (****)

A satellite is in orbit around a planet of mass M and radius r. At a given point in time, the satellite is at a distance $2r$ from the centre of the planet, and moving at a speed u, in a direction which makes an angle θ to the inward radial direction:



Given that in the course of its orbit, the satellite *just* grazes the surface of the planet, find the angle θ in terms of the initial speed u, the mass of the planet and the gravitational constant.

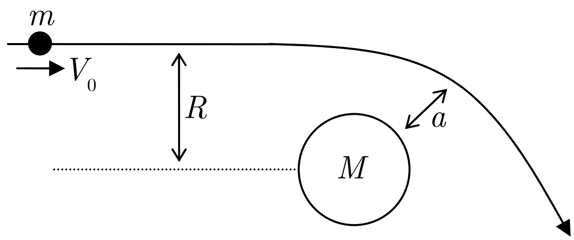
Now imagine that u was greater than the satellite's escape velocity, and that after grazing the planet, the satellite shoots off towards infinity:



Find the distance marked b in the diagram, which is the distance of the normal between the line of the satellite's final path and the centre of the planet.

Answers: $\sin\theta = \frac{1}{2}\sqrt{1+\dfrac{GM}{ru^2}}$ and $b = 2ur\sin\theta\sqrt{\left(1-\dfrac{GM}{ru^2}\right)^{-1}} = r\sqrt{\left(1+\dfrac{GM}{ru^2}\right)\left(1-\dfrac{GM}{ru^2}\right)^{-1}}$

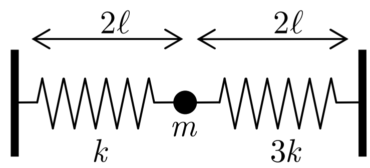
Question 35 (****)



A mass m, travelling at speed V_0 in a straight line, is deflected when it passes near a black hole of mass M which is at a perpendicular distance R from the original line of flight. Find a, the distance of closest approach between the mass and the black hole.

Answer: $a = R\left(1+\dfrac{2GM}{aV_0^2}\right)^{-1/2}$

Question 36 (****)



A particle of mass m is attached to two springs, of spring constants k and $3k$ and both of un-stretched lengths ℓ (see diagram above). The particle is released from rest from the position in the diagram above ($x = 0$). Find the resulting $x(t)$.

Answer: $x(t) = \ell\left[1 - \cos\left(2t\sqrt{\dfrac{k}{m}}\right)\right]$

Question 37 (****)

A thin uniform rod of length L and mass m is freely pivoted about its end. The rod is initially held horizontally and released from rest.

(A) What is the angular velocity of the rod about the pivot at the moment it reaches the vertical position?

When the rod is vertical, an impulse Q is applied to bring it to rest (this is in addition to any impulse provided by the pivot). Calculate

(B) The minimum impulse required to achieve this

(C) The impulse, and the distance from the pivot at which it must be applied, if there is to be no horizontal force at the pivot

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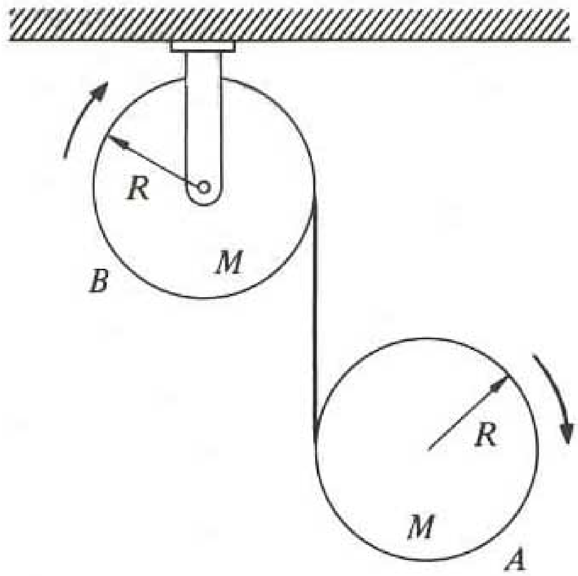
Answers:

(A) $\sqrt{3g / L}$

(B) $m\sqrt{gL / 3}$

(C) $m\sqrt{3gL / 4}$, applied $2L / 3$ from the pivot

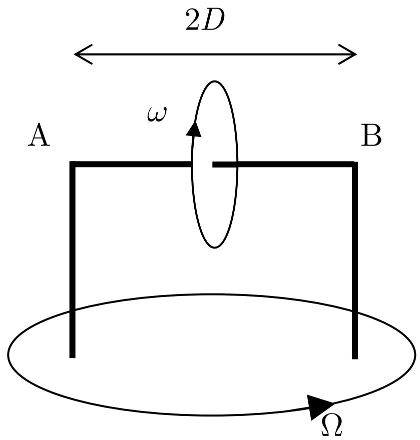
Question 38 (****)



A drum A of mass m and radius R is suspended from a drum B also of mass m and radius R, which is free to rotate about its axis. The suspension is in the form of a massless metal tape wound around the outside of each drum, and free to unwind. Gravity is directed downwards. Both drums are initially at rest. Find the initial acceleration of drum A, assuming that it moves straight down.

Answer: $a_A = \dfrac{2R^2 Mg}{2R^2 M + I}$

Question 39 (****)



A disc rotates at an angular velocity ω and is mounted on a turntable which rotates at a velocity Ω, as indicated in the diagram. Find the magnitude and direction of the forces on the wheel's axle at the points labelled A and B in the diagram.

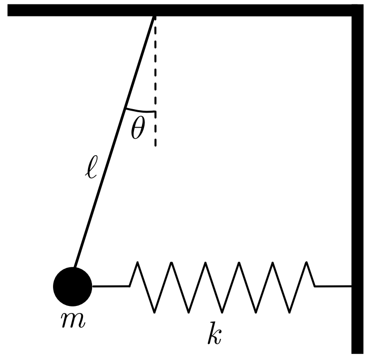
Answer: $F_A = \dfrac{mgD - I\omega\Omega}{2D}$ and $F_B = \dfrac{mgD + I\omega\Omega}{2D}$

Question 40 (****)

An artificial satellite is in a circular orbit around the moon at radius $R = \alpha r$, where r is the radius of the moon itself. A short burn of the satellite's motor provides an impulse which halves the satellite's speed without changing its direction, and this alters the orbit to one that just grazes the moon's surface. By considering the angular momentum and energy of the satellite at the apoapsis and periapsis of the new orbit, deduce the value of α.

Answer: $\alpha = 7$

Question 41 (****)



A simple pendulum consists of a point-like object of mass m attached to the end of a rod of negligible mass and length ℓ. A spring of negligible mass and force constant k is connected at one end to the point-like object and attached to a wall at the other end. The spring is relaxed when $\theta = 0$. The pendulum is displaced a **small** angle θ_0 from the vertical and released from rest. The system oscillates. You may approximate the direction of the spring force as horizontal throughout the motion.

Find the angular frequency of the oscillation (and check that when $g = 0$ and $\theta = 0$, the result reduces to other results you already know).

Answer: $\omega = \sqrt{\dfrac{g}{\ell} + \dfrac{k}{m}}$

Question 42 (**/*****)**

A disc of radius R and mass M is rotating at an angular velocity ω, when it is suddenly placed flat on a table with which it has a coefficient of kinetic friction μ_k. How long will the disc take to come to rest?

Hint: You will need to consider individual thin concentric rings in the disc, and integrate

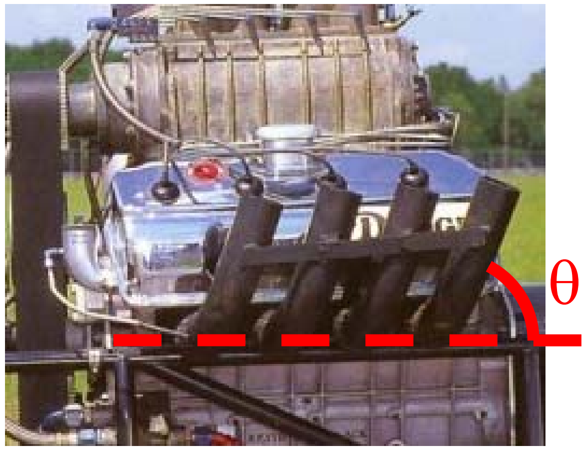
Answer: $t = 3\omega R \,/\, 4g\mu_k$

Question 43 (*****)

A circular coin of radius a is at rest and falls at speed u onto a smooth horizontal table. The perpendicular to its face makes an angle θ with the vertical. Determine the state of motion of the coin just after it strikes the table, assuming that the collision is elastic. Show that, when θ is small, the coin strikes the table a second time, and find the angle θ' at which it does so. You may assume that the system is moving so fast that gravity can be neglected.

Answer: $\theta' = \frac{5}{11}\theta$

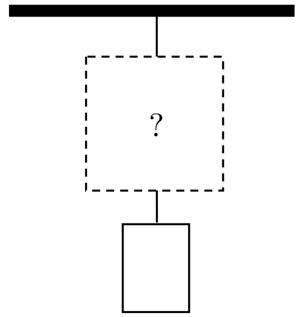
Question 44 (*****)



Dragsters are capable of using the thrust from their exhaust to increase their acceleration. Assuming that the dragster weights 1000 kg and that the tires have a coefficient of friction μ with the (specially treated) road surface, what is the optimal angle θ with the horizontal at which to align the exhaust pipes (as shown above) in order to give the maximum acceleration? How large would the engine thrust have to be to give an acceleration of $4.7g$ if $\mu = 2.5$?

Answer: 68.2°, 8000 N

Question 45 (*****)



In the figure above, a mass hangs from the ceiling. A piece of paper is help up to obscure three strings and two springs; all you see is two other string protruding from behind the paper, as shown. How should the three strings and two springs be attached to each other and to the two visible strings (different items can only be attached at their endpoints) so that if you start with the system at equilibrium and cut a certain one of the hidden strings, the mass will **rise up**?

Taken from Morin, CUP, 2007. Exercise 4.21.