# Hypothesis Testing

If the statistic is $>$ **critical value** or $<$ **− critical value**, reject $H_0$. Else, don't.

| Test type | $H_0$ (Null hyp.) | Test statistic | Two-tailed | | One-tailed | | Example |
|---|---|---|---|---|---|---|---|
| | | | $H_1$ | Critical value | $H_1$ | Critical value | |
| Single mean $n > 30$ or $\mathcal{N}$ | $\mu = a$ | $\dfrac{\bar{X} - a}{s \,/\, \sqrt{n}}$ | $\mu \neq a$ | $z_{(1-\alpha)/2}$ | $\mu < a$ *or* $\mu > a$ | $z_{1-\alpha}$ | "A sample of $n$ CBS students are found to have a mean IQ of $\bar{X}$, with sample standard deviation of $s$. Test, at the $\alpha$ % level, the statement that the mean IQ at CBS is $a$" |
| Single mean $n < 30$ | | | | $t_{n-1,(1-\alpha)/2}$ | | $t_{n-1,1-\alpha}$ | |
| Difference between two means (matched) | $\mu_X - \mu_Y = 0$ | $\dfrac{\bar{X} - \bar{Y}}{s_D \,/\, \sqrt{n}}$ | $\mu_1 - \mu_2 \neq 0$ | $z_{(1-\alpha)/2}$ | $\mu_1 - \mu_2 > 0$ *or* $< 0$ | $z_{1-\alpha}$ | "A sample of $n$ CBS students are found to have mean IQ $\bar{X}$ when they enter CBS, and mean IQ $\bar{Y}$ when they leave. Differences are calculated for each student and are found to have sample standard deviation $s_D$. Test, at the $\alpha$ % level, the statement that an MBA doesn't alter IQ." |
| Difference between two means (indep.) | | $\dfrac{\bar{X} - \bar{Y}}{\sqrt{\frac{s_X^2}{n_X} + \frac{s_Y^2}{n_Y}}}$ | | | | | "A sample of $n_X$ CBS students are found to have mean IQ $\bar{X}$ with standard deviation $s_X$. A sample of $n_Y$ Wharton students have $\bar{Y}$ and $s_Y$. Test, at the $\alpha$ % level, the statement that MBAs from CBS and Wharton have the same mean IQ" |

For a test with test statistic $z_0$, the two-tailed $P$-value is $2\left[1 - \mathbb{P}(Z \leq z_0)\right]$ and the one-tailed $P$-value is $\left[1 - \mathbb{P}(Z \leq z_0)\right]$ (replace $Z$ with $t_{n-1}$ for $n < 30$). Reject $H_0$ if the $P$-value is $\leq \alpha$.

Questions, corrections – email guetta@cantab.net