Example of Gomory's Cutting Plane Method

Consider the linear program

$$\min \quad 2x_1 + 15x_2 + 18x_3$$

Subject to

$$-x_1 + 2x_2 - 6x_3 \le -10$$
$$x_2 + 2x_3 \le 6$$
$$2x_1 + 10x_3 \le 19$$
$$-x_1 + x_2 \le -2$$
$$x_1, x_2, x_3 \ge 0$$

We can solve this problem the dual simplex method algorithm. The final tableau is as follows:

	x_1	x_2	x_3	z_1	z_2	z_3	z_4	
x_1	1	10	0	5	0	3	0	7
z_2	0	5	0	2	1	1	0	5
x_3	0	–2	1	–1	0	–½	0	½
z_4	0	11	0	5	0	3	1	5
	0	31	0	8	0	3	0	–23

Which corresponds to the solution

$$x_1 = 7 \qquad x_2 = 0 \qquad x_3 = \tfrac{1}{2} \qquad \text{Objective function} = 23$$

Now, imagine that we need x_1, x_2 and x_3 to be integers. The third row reads

$$-2x_2 + x_3 - z_1 - \tfrac{1}{2}z_3 = \tfrac{1}{2}$$
$$-2x_2 + x_3 - z_1 - \left\lceil \tfrac{1}{2} \right\rceil z_3 \le \tfrac{1}{2}$$
$$-2x_2 + x_3 - z_1 - z_3 \le \tfrac{1}{2}$$

If all the variables are integers, we also have that

$$-2x_2 + x_3 - z_1 - z_3 \le \left\lfloor \tfrac{1}{2} \right\rfloor$$
$$-2x_2 + x_3 - z_1 - z_3 \le 0$$

We can add this new inequality to our tableau in the form

$$-2x_2 + x_3 - z_1 - z_3 + z_5 = 0$$

Inserting this inequality into our tableau, we obtain

	x_1	x_2	x_3	z_1	z_2	z_3	z_4	z_5	
x_1	1	10	0	5	0	3	0	0	7
z_2	0	5	0	2	1	1	0	0	5
x_3	0	–2	1	–1	0	–½	0	0	½
z_4	0	11	0	5	0	3	1	0	5
z_5	0	–2	1	–1	0	–1	0	1	0
	0	31	0	8	0	3	0	0	–23

The matrix in the body of the tableau is $A_B^{-1}A$. Thus, those columns corresponding to basis variables should give $A_B^{-1}A_B = I$. Looking at the highlighted basis columns above, this is clearly not the case in the x_3 column. We fix this by subtracting the x_3 row from the last row:

	x_1	x_2	x_3	z_1	z_2	z_3	z_4	z_5	
x_1	1	10	0	5	0	3	0	0	7
z_2	0	5	0	2	1	1	0	0	5
x_3	0	–2	1	–1	0	–½	0	0	½
z_4	0	11	0	5	0	3	1	0	5
z_5	0	0	0	0	0	–½	0	1	–½
	0	31	0	8	0	3	0	0	–23

z_5 is negative, and we therefore pivot on that row. The only column with a negative entry in that row is z_3, so we pivot there.

	x_1	x_2	x_3	z_1	z_2	z_3	z_4	z_5	
x_1	1	10	0	5	0	0	0	6	4
z_2	0	5	0	2	1	0	0	2	4
x_3	0	–2	1	–1	0	0	0	–1	1
z_4	0	11	0	5	0	0	1	6	2
z_5	0	0	0	0	0	1	0	–2	1
	0	31	0	8	0	0	0	6	–26

This solution is both primal and dual optimal, with only integer solution.

$$x_1 = 4$$
$$x_2 = 0 \qquad \text{Objective function} = 26$$
$$x_3 = 1$$