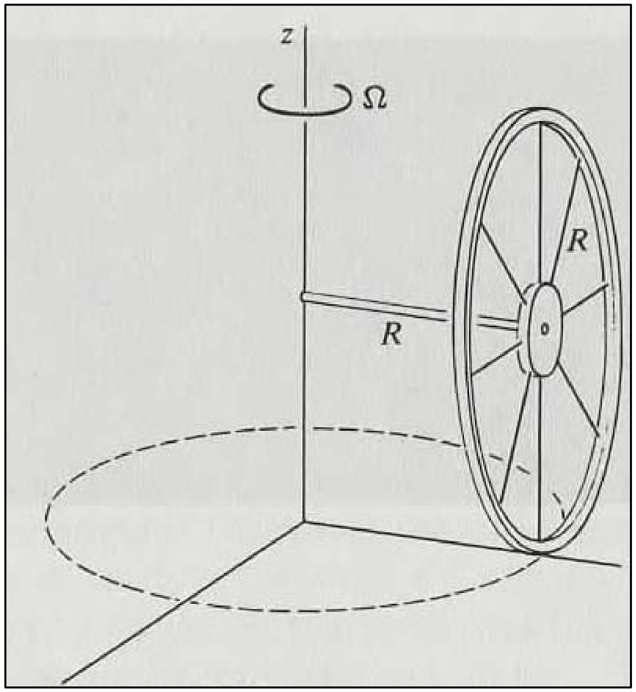
8.01 Review Session

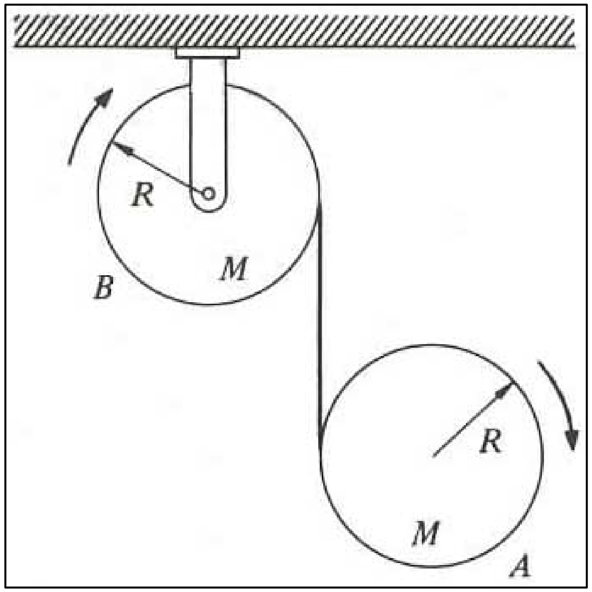
7$^{\text{th}}$ December 2008

Problem 1



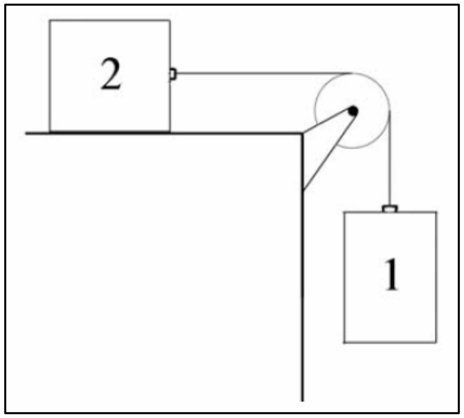
A thin hoop of mass m and radius r rolls without slipping about the z-axis. It is supported by an axle of length R through its centre. The hoop circles around the z-axis with angular speed Ω. [Note: the moment of inertia of a hoop for an axis about its diameter is $mR^2/2$] **What is the angular momentum of the wheel about a point where the axle meets the z-axis?**

Problem 2



A drum A of mass m and radius R is suspended from a drum B also of mass m and radius R, which is free to rotate about its axis. The suspension is in the form of a massless metal tape wound around the outside of each drum, and free to unwind. Gravity is directed downwards. Both drums are initially at rest. **Find the initial acceleration of drum A**, assuming that it moves straight down.

Problem 3

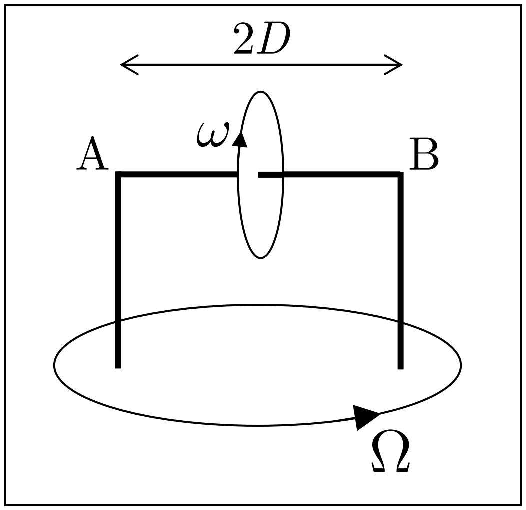


A pulley of mass m_p, radius R, and moment of inertia about its centre of mass I_{cm} is attached to the edge of a table. An inextensible string of negligible mass is wrapped around the pulley and attached to one end to block 1 that hangs over the edge of the table. The other end of the string is attached to block 2 which slides along a table. The coefficient of sliding friction between the table and block 2 is μ_k . Block 1 has mass m_1 and block 2 has mass m_2, with $m_1 > \mu_k m_2$. At time $t = 0$, the blocks are released from rest. At time $t = t_1$, block 1 hits the ground. **Find the acceleration of block 1, and the distance which it fell before it hit the ground**.

Problem 4

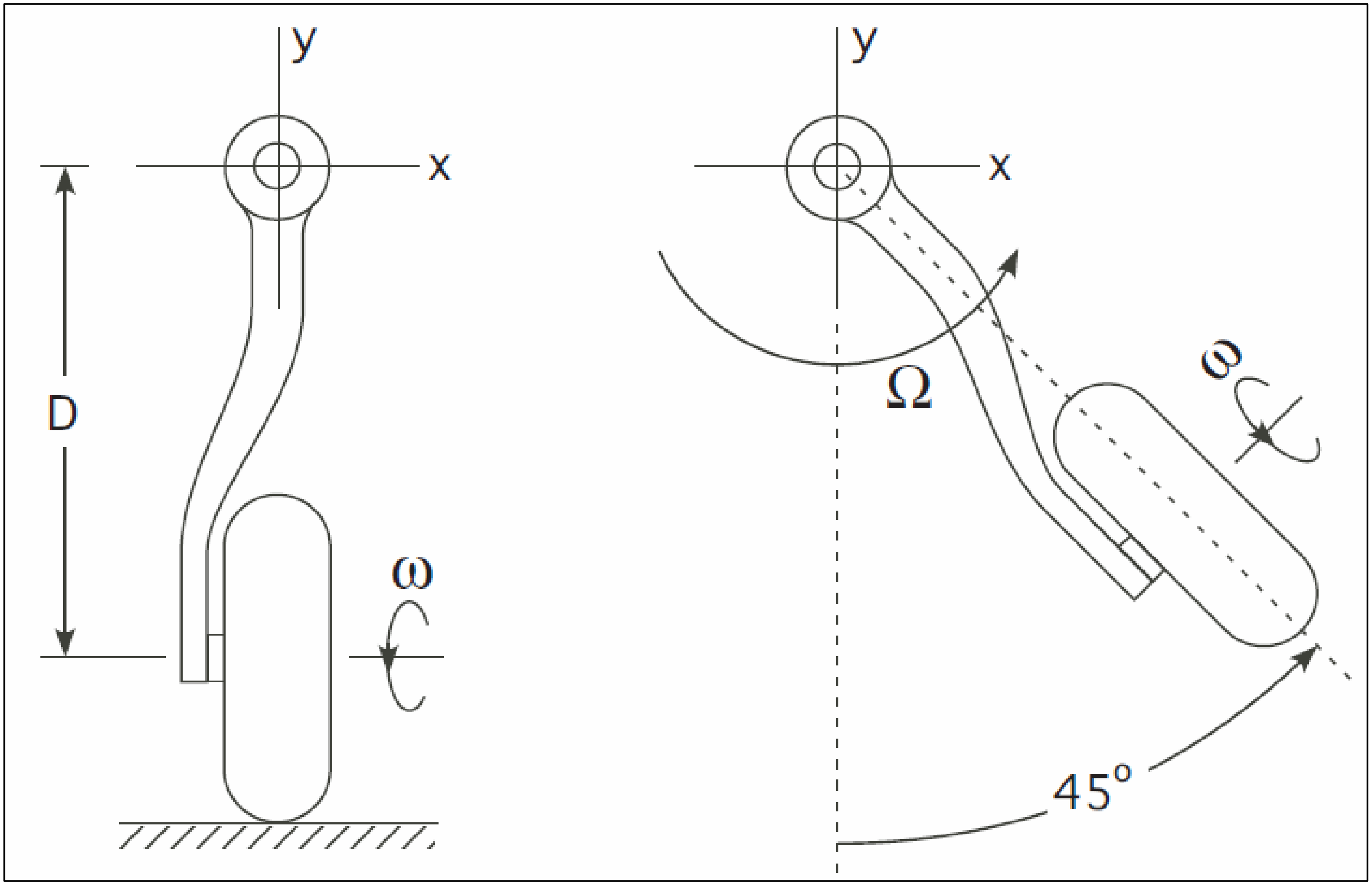
A uniform rod of length L and mass M which is free to rotate on a pivot at its top end is hanging vertically at rest. A bullet of mass m which is travelling horizontally at speed v strikes the lower end of the rod and is brought to rest, so falling to the ground. **Find the initial angular speed of the rod.**

Problem 5



A disc rotates at an angular velocity ω and is mounted on a turntable which rotates at a velocity Ω, as indicated in the diagram. **Find the magnitude and direction of the forces on the wheel's axle at the points labelled *A* and *B* in the diagram.**

Problem 6 (*part I*)



Problem 6 (*part II*)

The figure shows a landing gear being retracted by rotation about the z axis (which points out of the page in this figure). The wheel spins with a constant angular frequency ω as it is being retracted. When the assembly is at an angle of 45 degrees it is rotating about the z axis at a constant angular frequency Ω. Neglect all masses except that of the wheel, which has mass M. The moment of inertia of the wheel is I_0 about its axle and I_d about its diameter. **Find the angular momentum of the system about the origin and the torque that must be applied by the bearing at the origin when the assembly is at 45 degrees.** Express your answer as a vector with components along the x, y and z directions, and verify that it is still correct as $\omega \to 0$.

Problem 7

An artificial satellite is in a circular orbit around the moon at radius $R = \alpha r$, where r is the radius of the moon itself. A short burn of the satellite's motor provides an impulse which halves the satellite's speed without changing its direction, and this alters the orbit to one that just grazes the moon's surface. By considering the angular momentum and energy of the satellite at the apoapsis and periapsis of the new orbit, **deduce the value of $\boldsymbol{\alpha}$.**

[Check your answer: $\alpha = 7$]

Problem 8

A particle of mass m moves under an attractive central force of magnitude $F = br^3$. The angular momentum of the mass is equal to L.

a) Find the effective potential energy and make sketch of effective potential energy as a function of r.
b) Indicate on a sketch of the effective potential the total energy for circular motion.
c) Now assume that the radius of the particle's orbit varies between r_0 and $2r_0$. Find r_0.