Massachusetts Institute of Technology

Department of Physics

8.01 ~ Final Exam Equation Summary

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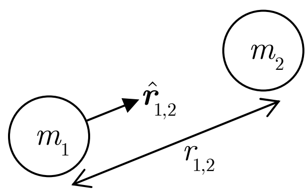
1. Common Interactions

Universal Law of Gravity

$$\vec{F}_{1,2} = -G\frac{m_1 m_2}{r_{1,2}}\hat{r}_{1,2}$$

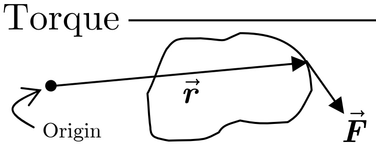
(*attractive*)



Near the surface of the earth, $|\boldsymbol{F}| = mg$, directed towards the earth's surface.

Torque

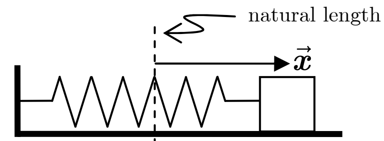
$$\vec{\tau} = \vec{r} \times \vec{F}$$



Hooke's Law

$$\vec{F} = -k\vec{x}$$

(*restoring*)



Friction

- Static friction: $0 \le f_s \le \mu_s N$
- Kinetic friction: $f_k = \mu_k N$ opposes motion.

2. Systems of Particles & Rigid Bodies

Centre of Mass (CoM)

$$\vec{R}_{\text{cm}} = \frac{1}{m_{\text{total}}}\sum_i m_i \vec{r}_i = \frac{1}{m_{\text{total}}}\int \vec{r}\,\mathrm{d}m$$

- $\vec{R}_{\text{cm}}$: Position vector of CoM
- $\sum_i$: Sum over all particles in system
- m_i: Mass of particle i
- m_{total}: Total mass of system
- $\vec{r}_i$: Position vector of particle i
- $\int \ldots \mathrm{d}m$: Integrate over rigid body

$$\vec{V}_{\text{cm}} = \frac{1}{m_{\text{total}}}\sum_i m_i \vec{v}_i = \frac{1}{m_{\text{total}}}\int \vec{v}\,\mathrm{d}m$$

- $\vec{V}_{\text{cm}}$: Velocity of CoM
- $\vec{v}_i$: Velocity of particle i

Moment of Inertia (MoI)

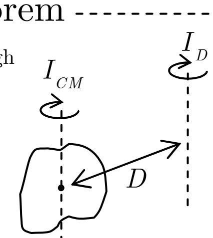
$$I = \sum_i m_i r_{\perp,i}^2 = \int r_{\perp,i}^2\,\mathrm{d}m$$

- I: MoI about a *particular* axis
- $\sum_i$: Sum over all particles in system
- $r_{\perp,i}$: *Perpendicular* distance of particle i to the axis

Parallel Axis Theorem

$$I_D = I_{CM} + mD^2$$

- I_D: MoI about an axis parallel to the axis through the CoM and a distance D away from it
- I_{CM}: MoI about an axis through the CoM
- m: Mass of rigid body



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3. Kinematics

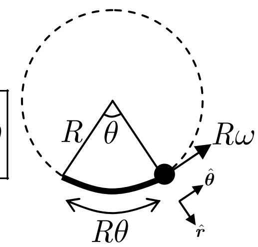
Definitions

- Linear: $\boxed{\vec{\boldsymbol{v}} = \frac{\mathrm{d}\vec{\boldsymbol{x}}}{\mathrm{d}t}}$ $\boxed{\vec{\boldsymbol{a}} = \frac{\mathrm{d}\vec{\boldsymbol{v}}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{\boldsymbol{x}}}{\mathrm{d}t^2}}$
- Rotational: $\boxed{\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}}$ $\boxed{\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}}$

Circular Motion

- Arc length: $\boxed{s = R\theta}$
- Velocity $\boxed{\vec{\boldsymbol{v}} = R\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{\boldsymbol{\theta}} = R\omega\hat{\boldsymbol{\theta}}}$
- Acceleration:

$$\boxed{\boldsymbol{a} = -\omega^2 R\hat{\boldsymbol{r}} + R\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\hat{\boldsymbol{\theta}} = -\frac{v^2}{R}\hat{\boldsymbol{r}} + R\alpha\hat{\boldsymbol{\theta}}}$$



4. Simple Harmonic Motion

Motion

$$\boxed{x(t) = A\cos(\omega t) + B\sin(\omega t)}$$

Definitions

- Period $\boxed{T = 2\pi\,/\,\omega}$
- Frequency $\boxed{f = 1\,/\,T}$

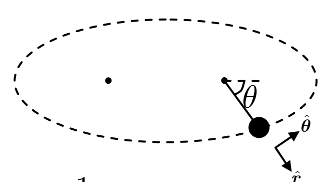
5. Central Force Motion

Two-body problem

Two bodies of masses m_1 and m_2 behave like a single body of mass μ orbiting around the common centre of mass, where

$$\boxed{\mu = \frac{m_1 m_2}{m_1 + m_2}}$$

Elliptical orbits



$$\begin{aligned} E &= \frac{1}{2}mv^2 + U(r) \\ &= \frac{1}{2}mv_{\hat{r}}^2 + \frac{1}{2}mv_{\hat{\theta}}^2 + U(r) \\ &= \frac{1}{2}mv_{\hat{r}}^2 + \frac{L^2}{2\mu r^2} + U(r) \\ &= K_{\text{eff}} + U_{\text{eff}} \end{aligned}$$

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6. Work & Energy

Kinetic Energy

- Linear: $\boxed{\frac{1}{2}mv^2}$
- Rotational: $\boxed{\frac{1}{2}I\omega^2}$

Work done by a Force

$$\boxed{W = \int_{\text{original position}}^{\text{final position}} \vec{\boldsymbol{F}} \cdot \mathrm{d}\vec{\boldsymbol{r}}}$$

Work-Energy Theorem

$$\boxed{U_{\text{initial}} + KE_{\text{initial}} = U_{\text{final}} + KE_{\text{final}} + W}$$

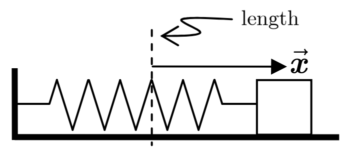
Potentials & Conservative Forces

$$\boxed{PE_{\text{at } r} = -\int_{\text{position of 0 potential}}^{r} \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}}$$

Potential Energy Functions

- Spring: $\boxed{PE = \frac{1}{2}k\left|\vec{\boldsymbol{x}}\right|^2}$

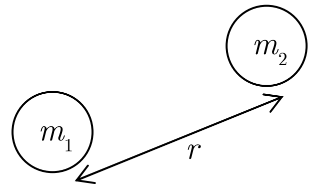
(With PE = 0 at equilibrium)



- Universal gravitation:

$$\boxed{PE = -G\frac{m_1 m_2}{r}}$$

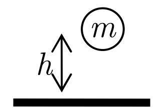
(With PE = 0 at $r = \infty$)



- Gravity near the earth's surface:

$$\boxed{PE = -mgh}$$

(With PE = 0 at the earth's surface)

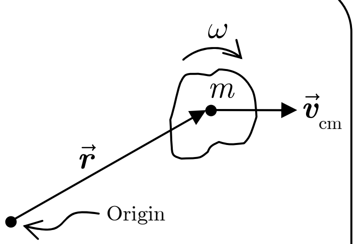


7. Dynamics

Angular Momentum

For a body translating and rotating about its centre of mass

$$\boxed{\vec{\boldsymbol{L}} = \underbrace{\vec{\boldsymbol{r}} \times m\vec{\boldsymbol{v}}_{\text{cm}}}_{\text{Translational motion}} + \underbrace{I_{\text{cm}}\vec{\boldsymbol{\omega}}}_{\text{Rotational motion}}}$$



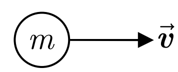
- For a point particle, only the translational part need be considered.
- For a body rotating but not translating, only the rotational part need be considered, but the moment of inertia I <u>must</u> be taken about the correct axis.

Equations of Motion

$$\boxed{\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \frac{\mathrm{d}\vec{\boldsymbol{L}}}{\mathrm{d}t}} \qquad \boxed{\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}} = \frac{\mathrm{d}\vec{\boldsymbol{p}}}{\mathrm{d}t}}$$

Linear Momentum

$$\boxed{\vec{\boldsymbol{p}} = m\vec{\boldsymbol{v}}}$$



Linear Impulse

$$\boxed{\vec{\boldsymbol{I}} = \int_{\text{start time}}^{\text{end time}} \vec{\boldsymbol{F}}\,\mathrm{d}t}$$

Angular Impulse

$$\boxed{\vec{\boldsymbol{J}} = \int_{\text{start time}}^{\text{end time}} \vec{\boldsymbol{\tau}}\,\mathrm{d}t = \int_{\text{start time}}^{\text{end time}} \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}}\,\mathrm{d}t = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{I}}}$$

Impulse/Momentum Principle

$$\boxed{\vec{\boldsymbol{I}} = \vec{\boldsymbol{p}}_{\text{after impulse}} - \vec{\boldsymbol{p}}_{\text{before impulse}} = \Delta\vec{\boldsymbol{p}}}$$

$$\boxed{\vec{\boldsymbol{J}} = \vec{\boldsymbol{L}}_{\text{after impulse}} - \vec{\boldsymbol{L}}_{\text{before impulse}} = \Delta\vec{\boldsymbol{L}}}$$

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