Physics 8.01 T – Section L05

Solutions to Quiz 1

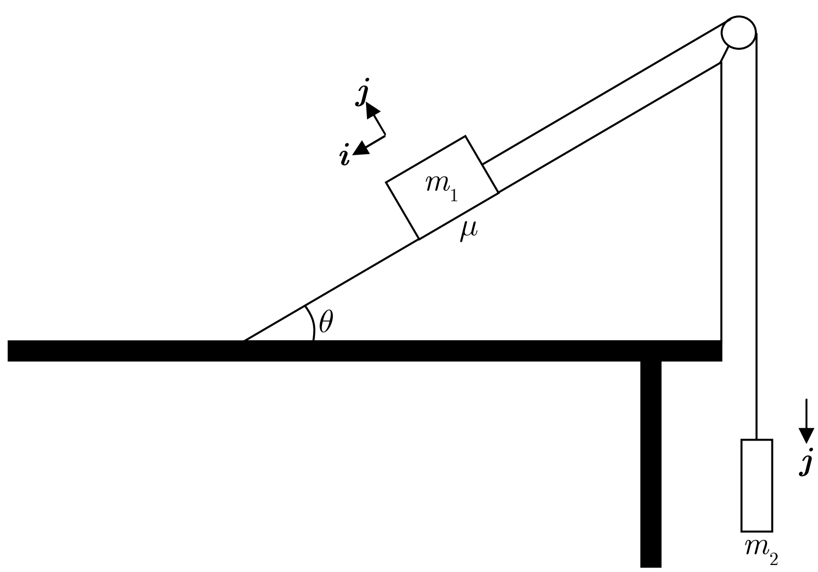
Part (a)

I need to find out what the acceleration of block 1 is. Once I know that, I'll easily be able to work out how far it travels, because the acceleration's constant. And to find its acceleration using $F = ma$, I'll need to know something about the forces acting on it. So free-body diagrams seem to be the way to go.

That said, my strategy will be as follows:

1. I'll choose a coordinate system
2. I'll draw free-body diagrams for both blocks
3. I'll apply all the facts I know about the forces – including $f = ma$
4. I'll tidy up my result to find the acceleration
5. I'll use the constant-acceleration equations to find the distance travelled

Step 1 It seems most sensible to use different coordinate systems for each block. The most sensible ones are:



With the origin at the block.

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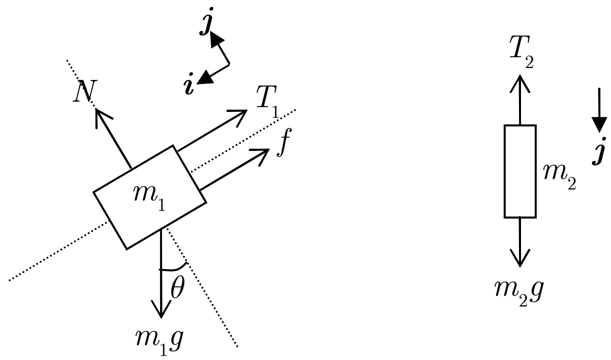
Step 1 A free body diagram for each block is



It makes sense that the friction on block 1 goes up, because we expect it to move down…

Step 3 What do I know about the quantities in the problem?

- The tensions on each block are Newton-Pairs, and so

$$\boxed{T_1 = T_2}$$

- The expression for friction is

$$\boxed{f = \mu N}$$

- Time to resolve the forces for each block
  - For block 2:

In the j direction

F_{2j}	$=$	$M_2 a_{2j}$
$m_2 g - T_2$	$=$	$m_2 a_{2j}$

  - For block 1:

In the i direction

F_{1i}	$=$	$m_1 a_{1i}$
$m_1 g \sin\theta - T_1 - f$	$=$	$m_1 a_{1i}$

In the j direction

F_{1j}	$=$	$m_1 a_{1j}$
$N - m_1 g \cos\theta$	$=$	$m_1 a_{1j}$

- What do we know about the accelerations?
  - We know that block 1 is not moving off the plane, so

$$\boxed{a_{1j} = 0}$$

- If block 1 moves down (positive displacement) block 2 must move up because it is attached to the string (negative displacement), so

$$\boxed{a_{1i} = -a_{2j}}$$

Step 4 From the tables above, we have three equations:

$$m_2 g - T_2 = m_2 a_{2j}$$
$$m_1 g \sin\theta - T_1 - f = m_1 a_{1i}$$
$$N - m_1 g \cos\theta = m_1 a_{1j}$$

First, we can set $\boxed{a_{1j} = 0}$

$$m_2 g - T_2 = m_2 a_{2j}$$
$$m_1 g \sin\theta - T_1 - f = m_1 a_{1i}$$
$$N - m_1 g \cos\theta = 0$$

Next, we can eliminate a_{2j} because $\boxed{a_{1i} = -a_{2j}}$:

$$m_2 g - T_2 = -m_2 a_{1i}$$
$$m_1 g \sin\theta - T_1 - f = m_1 a_{1i}$$
$$N - m_1 g \cos\theta = 0$$

We can then eliminate T_2 because $\boxed{T_1 = T_2}$

$$m_2 g - T_1 = -m_2 a_{1i}$$
$$m_1 g \sin\theta - T_1 - f = m_1 a_{1i}$$
$$N - m_1 g \cos\theta = 0$$

We can substitute $\boxed{f = \mu N}$

$$m_2 g - T_1 = -m_2 a_{1i}$$
$$m_1 g \sin\theta - T_1 - \mu N = m_1 a_{1i}$$
$$N - m_1 g \cos\theta = 0$$

From the last equation, we can find out that $N = m_1 g \cos\theta$ and put that into the second equation:

$$m_2 g - T_1 = -m_2 a_{1i}$$
$$m_1 g \sin\theta - T_1 - \mu m_1 g \cos\theta = m_1 a_{1i}$$

From the first equation, we can find that $m_2 g + m_2 a_{1i} = T_1$ and put that into the second equation:

$$m_1 g \sin\theta - \left(m_2 g + m_2 a_{1i}\right) - \mu m_1 g \cos\theta = m_1 a_{1i}$$

And now it's just a question of tidying up:

$$m_1 g\sin\theta - m_2 g - m_2 a_{1i} - \mu m_1 g\cos\theta = m_1 a_{1i}$$
$$m_1 g\sin\theta - m_2 g - \mu m_1 g\cos\theta = m_1 a_{1i} + m_2 a_{1i}$$
$$m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g = \left(m_1 + m_2\right) a_{1i}$$
$$\boxed{a_{1i} = \frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2}}$$

Step 5 The acceleration from then on will remain constant, and so we can use the constant-acceleration equations. Let's see what we have and what we want

$\boldsymbol{v_0}$	✓ – originally, the block is released from rest. So $v_0 = 0$
$\boldsymbol{v_f}$	
$\boldsymbol{a}$	✓ – we just found that
$\boldsymbol{x}$	This is what we want to find
$\boldsymbol{t}$	✓ – We're told me want to find x after t has elapsed

The correct equation to use here is therefore

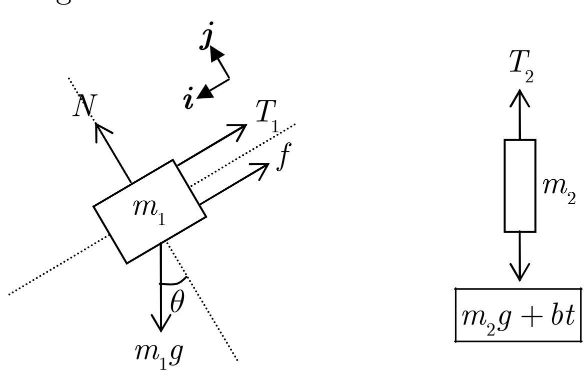
$$x = v_0 t + \tfrac{1}{2} a t^2$$

Putting in our values

$$x = \frac{1}{2} a t^2$$
$$\boxed{x = \frac{1}{2}\left(\frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2}\right)t^2}$$

Part B

The new free-body diagrams are:



The one force that's changed is the force that pulls the second block down (I've put it in a box above).

It is good practice to go through the entire mechanism above again – the answer that comes out is

$$a_{1i} = \frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - \left(bt + m_2 g\right)}{m_1 + m_2}$$
$$= \frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2} - \frac{b}{m_1 + m_2} t$$

When the block stops, its **velocity** is 0 (*not* its acceleration, as some of you thought – think about a ball you throw up in the air; when it's at rest at the top of its journey and it changes direction, it's acceleration is still g, not 0). To find the velocity, we need to integrate the acceleration with respect to time:

$$v_{1i} = \int \frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2} - \frac{b}{m_1 + m_2} t \, \mathrm{d}t$$
$$= \frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2} t - \frac{b}{2\left(m_1 + m_2\right)} t^2 + C$$

At $t = 0$, the block is at rest and so $v = 0$. Therefore, $C = 0$

$$v_{1i} = \frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2} t - \frac{b}{2\left(m_1 + m_2\right)} t^2$$

We need to find the time when the velocity is 0.

$$\frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2} t - \frac{b}{2\left(m_1 + m_2\right)} t^2 = 0$$

We can divide by t – we know that $t = 0$ is a solution:

$$\frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2} - \frac{b}{2\left(m_1 + m_2\right)} t = 0$$

And now we just need to solve

$$\frac{m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g}{m_1 + m_2} = \frac{b}{2\left(m_1 + m_2\right)} t$$

$$\boxed{t = \frac{2}{b}\left[m_1 g\left(\sin\theta - \mu\cos\theta\right) - m_2 g\right]}$$

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